Given: $\triangle A B C$; point $D$ is on $\overline{B C}$ such that $\overline{A D}$ is an angle bisector of $\triangle A B C$.
Prove: $\frac{B D}{C D}=\frac{B A}{C A}$
Plan: Draw line $\ell$ through point $C$ parallel to $\overline{A D}$.
Extend $\overline{B A}$ past $A$ to intersect line $\ell$ at $P$.

## Statement

## Reason



1. $\triangle A B C$; point $D$ is on $\overline{B C}$ such that
2. Given
$\overline{A D}$ is an angle bisector of $\triangle A B C$.
3. Draw line $\ell$ through $C \|$ to $\overline{A D}$. 2.
4. Extend $\overline{B A}$ to intersect line $\ell$ at $P$.
5. $\frac{B D}{C D}=\frac{B A}{P A}$
6. 
7. $\angle C P A \cong \angle D A B$
8. 
9. $\angle D A B \cong \angle D A C$
10. 
11. $\angle D A C \cong \angle P C A$
12. 
13. $\angle C P A \cong \angle P C A$
14. 
15. $\overline{P A} \cong \overline{C A}$
16. 
17. $\frac{B D}{C D}=\frac{B A}{C A}$
18. 

If $\overline{A D}$ is an angle bisector of $\triangle A B C$, then


Ex: Triangle $A B C$ has sides of 8,12 and 15 . Point $P$ is on $\overline{B C}$ so that $\overline{A P}$ is an angle bisector of the triangle. Find the length of $\overline{B P}$.


1. In $\triangle A B C, D$ is on $\overline{B C}$ so that $\overline{A D}$ is an angle bisector. If $A B=10, B D=6$ and $D C=8$, find the length of side $\overline{A C}$.
2. In $\triangle A B C, D$ is on $\overline{B C}$ so that $\overline{A D}$ is an angle bisector. If $A B=12, B C=18$ and $A C=15$, find the lengths of $\overline{B D}$ and $\overline{D C}$.
3. Triangle $O P Q$ has vertices $O(0,0), P(5,12)$ and $Q(14,0)$.
a. find the lengths of all three sides of $\triangle O P Q$.
b. If $R$ has coordinates $R(x, 0)$, find the value of $x$ so that $\overline{P R}$ bisects $\angle O P Q$.
4. In right $\triangle A B C, C$ is the right angle, $A C=6$ and $B C=8$. Points $P$ and $Q$ and $M$ are on hypotenuse $\overline{A B}$ such that $\overline{C P}$ is an altitude of the triangle, $\overline{C Q}$ is an angle bisector of the triangle and $\overline{C M}$ is a median of the triangle.
a. Find the length of $\overline{A B}$.
b. Find the length of $\overline{A P}$.
c. Find the length of $\overline{C P}$.
d. Find the length of $\overline{A Q}$.
e. Find the length of $\overline{C M}$.*
f. Find the measure of $\angle A$.
g. Find the measure of $\angle A Q C$.
h. Find the measure of $\angle A M C$.

*On a homework assignment a long time ago, you learned that the median to the hypotenuse of a right triangle is half the length of the hypotenuse.
5. A right triangle has legs of length 30 and 40.
a. Find the lengths of the two parts of the hypotenuse created by the altitude.
b. Find the length of the altitude to the hypotenuse.
6. Find the value of $x$ in the diagram algebraically.

