Geometry Notes S - 10: Angle Bisector Theorem

Reason

Given: $\triangle ABC$; point *D* is on \overline{BC} such that \overline{AD} is an angle bisector of $\triangle ABC$. Prove: $\frac{BD}{CD} = \frac{BA}{CA}$

Plan: Draw line ℓ through point *C* parallel to \overline{AD} . Extend \overline{BA} past *A* to intersect line ℓ at *P*.

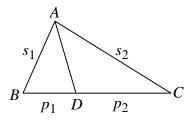
<u>Statement</u>

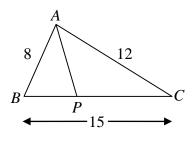
1. $\triangle ABC$; point <i>D</i> is on <i>BC</i> such that	1. Given
\overline{AD} is an angle bisector of $\triangle ABC$.	
2. Draw line ℓ through $C \parallel$ to \overline{AD} .	2.
3. Extend \overline{BA} to intersect line ℓ at <i>P</i> .	3.
4. $\frac{BD}{CD} = \frac{BA}{PA}$	4.
5. $\angle CPA \cong \angle DAB$	5.
6. $\angle DAB \cong \angle DAC$	6.
7. $\angle DAC \cong \angle PCA$	7.

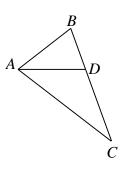
- 8. $\angle CPA \cong \angle PCA$ 8.
- 9. $\overline{PA} \cong \overline{CA}$ 9.
- 10. $\frac{BD}{CD} = \frac{BA}{CA}$ 10.

If \overline{AD} is an angle bisector of $\triangle ABC$, then

Ex: Triangle *ABC* has sides of 8, 12 and 15. Point *P* is on \overline{BC} so that \overline{AP} is an angle bisector of the triangle. Find the length of \overline{BP} .







1. In $\triangle ABC$, D is on \overline{BC} so that \overline{AD} is an angle bisector. If AB = 10, BD = 6 and DC = 8, find the length of side \overline{AC} .

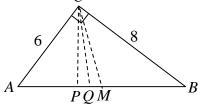
2. In $\triangle ABC$, D is on \overline{BC} so that \overline{AD} is an angle bisector. If AB = 12, BC = 18 and AC = 15, find the lengths of \overline{BD} and \overline{DC} .

- 3. Triangle OPQ has vertices O(0, 0), P(5, 12) and Q(14, 0).
 - a. find the lengths of all three sides of $\triangle OPQ$.
 - b. If R has coordinates R(x, 0), find the value of x so that \overline{PR} bisects $\angle OPQ$.

- 4. In right $\triangle ABC$, C is the right angle, AC = 6 and BC = 8. Points P and Q and M are on hypotenuse \overline{AB} such that \overline{CP} is an altitude of the triangle, \overline{CQ} is an angle bisector of the triangle and \overline{CM} is a median of the triangle.
 - a. Find the length of \overline{AB} .
 - c. Find the length of \overline{CP} .
- b. Find the length of \overline{AP} .
- d. Find the length of \overline{AQ} .
- e. Find the length of \overline{CM} .*

g. Find the measure of $\angle AQC$.

- f. Find the measure of $\angle A$.
- h. Find the measure of $\angle AMC$.



*On a homework assignment a long time ago, you learned that the median to the hypotenuse of a right triangle is half the length of the hypotenuse.

- 5. A right triangle has legs of length 30 and 40.a. Find the lengths of the two parts of the hypotenuse created by the altitude.
 - b. Find the length of the altitude to the hypotenuse.

6. Find the value of x in the diagram algebraically.

