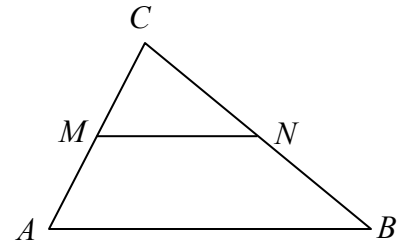# Geometry Notes S - 6: Midpoints and Parallel Lines

Theorem: If a segment joins the midpoints of two sides of a triangle then it is parallel to and half the length of the third side of the triangle.
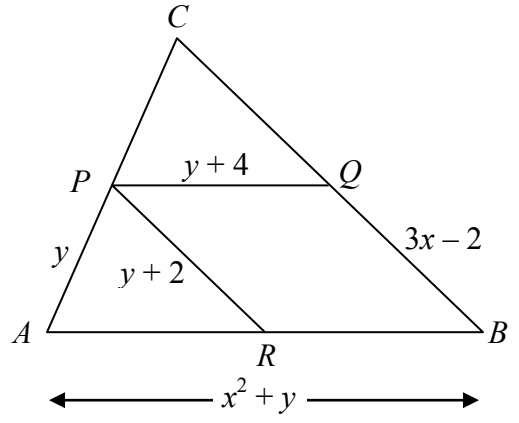
Given: $\triangle ABC$, M is the midpoint of $\overline{AC}$ and N is the midpoint of $\overline{BC}$.

Prove: a. $\dfrac{MN}{AB} = \dfrac{1}{2}$

   b. $\overline{MN} \parallel \overline{AB}$

Statement	Reason

Ex: In the diagram, P, Q and R are the midpoints of the sides of $\triangle ABC$. Find the perimeter of $\triangle ABC$.
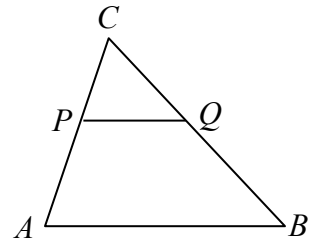
Theorem: If a line parallel to one side of a triangle intersects the other two sides, then
    a. it forms two similar triangles and
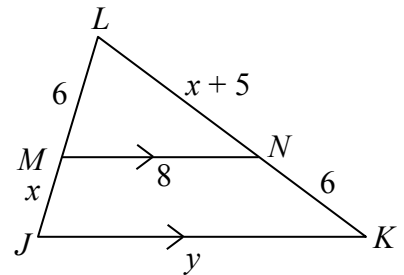   b. it divides the intersected sides in proportion.

If $\overline{PQ} \parallel \overline{AB}$, then

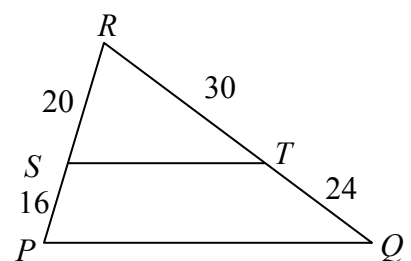$\triangle PQC \sim \triangle ABC$ and $\dfrac{CP}{PA} = \dfrac{CQ}{QB}$

Proof:

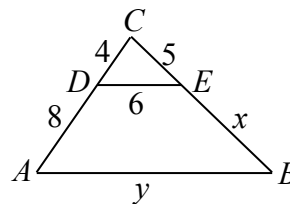Ex: Solve for x and y in the diagram at right.

Note: The converse of part b of the theorem is also true: If a line intersects two sides of a triangle and divides those sides in proportion, then it is parallel to the third side.
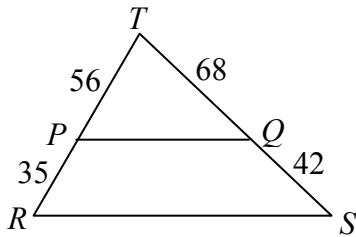
Ex: Is $\overline{PQ} \parallel \overline{ST}$?

1. Find the length of the line segment that joins the midpoints of the congruent sides of an isosceles triangle whose base measures 18.

2. In $\triangle ABC$, D, E and F are the midpoints of sides $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ respectively.
   a. If $FE = 7$, find the value of AB.
   b. If $BC = 17$, find the value of DF.
   c. If $ED = 3x - 2$, and $AC = 4x + 4$, find the numerical values of both ED and AC.

3. A segment joining the midpoints of two consecutive sides of a parallelogram measures 20. Find the length of one diagonal of the parallelogram.

4. In $\triangle ABC$, M, R, and T are the midpoints of sides $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$, respectively. If $AB = 22$, $BC = 12$, and $AC = 16$,
   a. Find the perimeter of $\triangle ABC$.
   b. Find the perimeter of $\triangle MRT$.

5. In the diagram at right $\overline{DE} \parallel \overline{AB}$. Find the values of x and y.

6. In the diagram at left, determine if $\overline{PQ}$ ∥ $\overline{AB}$ and justify your answer.

(diagram: triangle with apex T; angles 56 and 68 at T; point P with 35, point Q with 42; base R, S)

7. In the diagram at right, $\triangle MAN$ is isosceles with base $MN = 4$ and sides 6, $\overline{HT} \parallel \overline{MN}$ and $HT = 3$. Find the perimeter of quadrilateral $MHTN$.

(diagram: triangle with apex A, H and T on the sides, base M N)
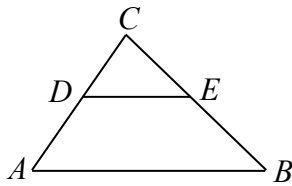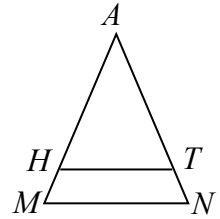
8. In the diagram at left, $\overline{DE} \parallel \overline{AB}$, $CE = x$, $AB = x + 15$ and $DE = EB = 6$. Find the numerical value of CE.

(diagram: triangle with apex C, D and E on the sides, base A B)

9. In rectangle $ABCD$, $AB = 8$ and $BC = 6$. E and F are on $\overline{AB}$ and $\overline{BC}$ such that $\overline{EF} \parallel \overline{AC}$. If $EF = 6.25$, find the length of EB.

10. a. The sides of two squares are in the ratio 2:3. What is the ratio of the *areas* of the squares?
    b. The sides of two squares are in the ratio $a:b$. What is the ratio of the *areas* of the squares?