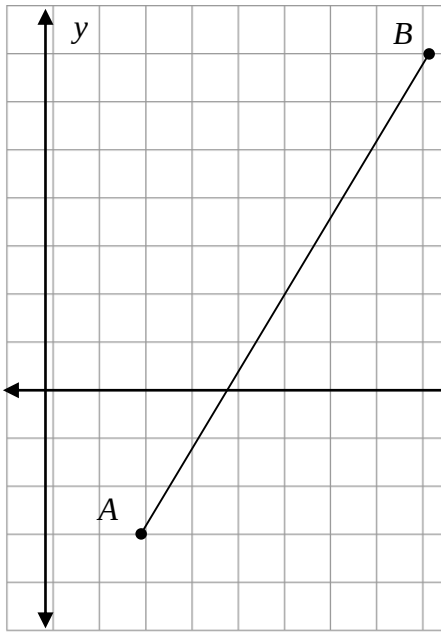
Geometry Notes CG - 7: Midpoint Formula

Ex: What is the midpoint of $\overline{AB}$ with $A(2, -3)$ and $B(8, 7)$?



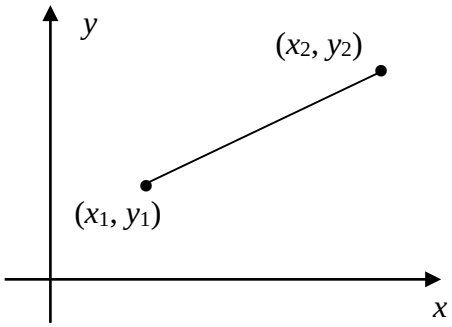
To find the middle of two numbers, say 74 and 92, we average them: $\frac{74 + 92}{2} = 83$. Do the same thing to find the midpoint of a line segment:

Average the x's: $\frac{2 + 8}{2} = 5$

Average the y's: $\frac{(-3) + 7}{2} = 2$

so midpoint is (5, 2)

Midpoint Formula

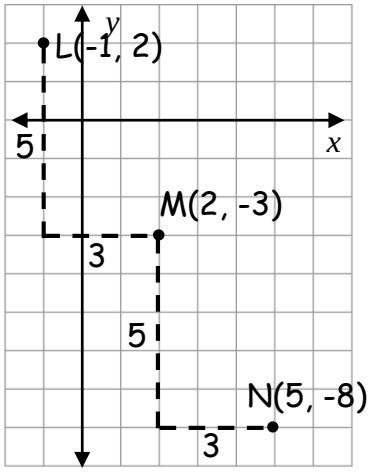


$$\text{Midpoint} = (\bar{x}, \bar{y}) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Ex: Find the midpoint of $\overline{RS}$ if $R(a, a + 2)$ and $S(3a, a - 8)$.

$$(\bar{x}, \bar{y}) = \left(\frac{a + 3a}{2}, \frac{a + 2 + a - 8}{2}\right) = (2a, a - 3) \text{ (ans)}$$

Ex: Find the coordinates of $N(x, y)$ if $M(2, -3)$ is the midpoint of $\overline{LN}$ and L has coordinates $(-1, 2)$.



1. Graphically

   From L to M is "down 5, right 3." Repeat to find N(5, -8)

2. Algebraically Let N have coordinates (x, y):

$$(\bar{x}, \bar{y}) = \left(\frac{-1 + x}{2}, \frac{2 + y}{2}\right) = (2, -3)$$

$$\frac{-1 + x}{2} = 2 \rightarrow x = 5$$

$$\frac{2 + y}{2} = -3 \rightarrow y = -8$$

So N(5, -8).

Summary of Formulas

1. Distance: $d = \sqrt{(\Delta x)^2 + (\Delta y)^2}$

   Answer: A non-negative number.

2. Slope: $m = \frac{\Delta y}{\Delta x}$

   Answer: A ratio (fraction) (although it may be expressed as a decimal number.)

3. Midpoint: Midpoint = $(\bar{x}, \bar{y})$

   Answer: An ordered pair.