Geometry Notes S - 1: Similarity

Review

Rigid motion (aka *isometry*): a composition of transformations that

Basic rigid motions: Translations, Rotations, Reflections

Major properties: 1. Preserves length

2. Preserves angle measure

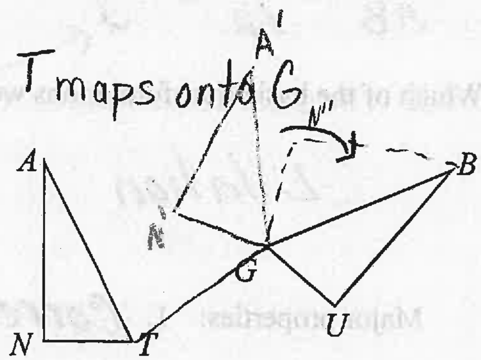
Congruent: two figures are congruent if there is a sequence of rigid motions that can map one onto the other.

Ex: a. Describe a sequence of basic rigid motions that could show $\Delta ANT \cong \Delta BUG$ (Remember, there is more than one correct answer.)

1) Translate ΔANT along vector $\overrightarrow{TG}$ until T maps onto G

2) Rotate $\Delta A'N'T'$ clockwise around pt G until $\overline{A'G}$ maps onto $\overline{BG}$

3) Reflect $\Delta GN''B$ over line $\overline{BG}$ until $\Delta GN''B$ maps onto ΔBUG



b. Are the two triangles congruent?

* $\Delta ANT \cong \Delta BUG$ b/c length and angle measure are preserv in Translations, Rotations and Reflections since we mapped ΔANT onto BUG using only these motions they are $\cong$.

Congruence is nicely defined in terms of rigid motions. But rigid motions are not a convenient method to actually decide if two figures are congruent. So for triangles, we developed some congruence theorems:

$\Delta \cong \Delta$ if: SAS

ASA

SSS

New

Similarity transformation: a composition of transformations that – changes size

Ex: $\triangle ABC$ has $AB = 12$ and $BC = 18$.

a. After the transformation T, $A'B' = 18$ and $B'C' = 24$. Is T a similarity transformation?

$\frac{new}{old} = \frac{A'B'}{AB}\ \frac{18}{12} = \frac{3}{2}$ $\quad \frac{B'C'}{BC} = \frac{24}{18} = \frac{4}{3}$ No ratio is not the same.

b. After the transformation S, $A'B' = 8$ and $B'C' = 12$. Is S a similarity transformation?

$\frac{A'B'}{AB} = \frac{8}{12} = \frac{2}{3}$ $\quad \frac{B'C'}{BC} = \frac{12}{18} = \frac{2}{3}$ Yes ratios are the same.

Which of the basic transformations we learned are similarity transformations?

Dilation

Major properties: 1. Corresponding angles are $\cong$

2. Corresponding sides are in proportion

Similar: two figures are similar if there is a sequence of transformations that will map one onto the other

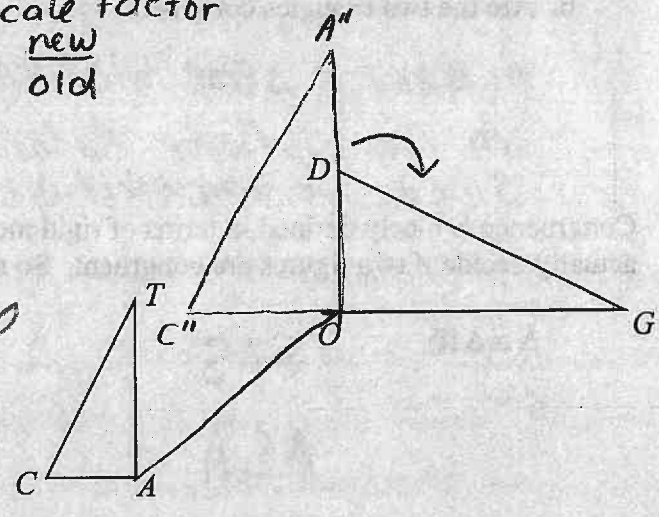
Ex: a. Describe a similarity transformation that could show $\triangle CAT \sim \triangle DOG$ (Note: There is more than one correct answer.)

scale factor $\frac{new}{old}$

1) Dilate $\triangle CAT$ by $r = \frac{DG}{CT}$

2) Translate $\triangle C'A'T'$ along vector $\vec{AO}$ until A' maps onto O

3) Rotate $\triangle C''A''T''$ clockwise until A'' maps onto G''



b. Are the two triangles similar?

Yes the $\triangle$s are similar b/c $\triangle CAT$ maps onto $\triangle DOG$ by a sequence of transformations

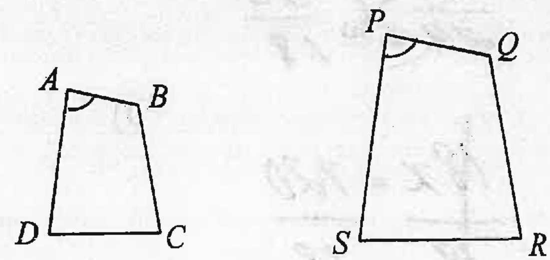
Properties of similar polygons:

a. All pairs of corresponding (matching) angles are $\cong$

b. All pairs of corresponding (matching) sides are in proportion

NOTE: Just like for congruent polygons, similarity statements are written so that corresponding vertices are in the ***same order.***

Ex: If $ABCD \sim PQRS$, then



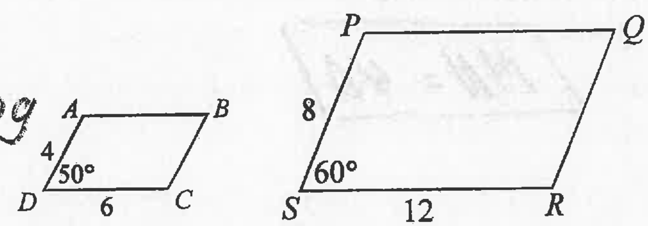
$\angle A \cong \angle P$ $\qquad \frac{AB}{PQ} \qquad \frac{BC}{QR}$

$\angle B \cong \angle Q$

$\angle C \cong \angle R$ $\qquad \frac{AD}{PS} \qquad \frac{DC}{SR}$

$\angle D \cong \angle S$

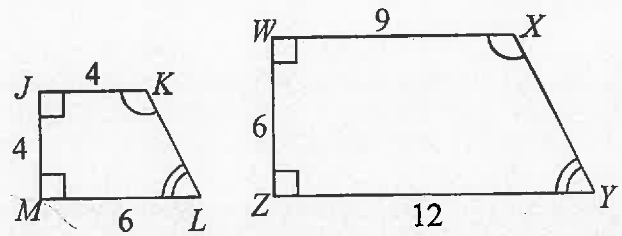
Ex: Is $\underbrace{ABCD \sim PQRS}$?



No because corresponding angles are not $\cong$

$\angle D \ncong \angle S$

Ex: Is $ABCD \sim PQRS$?

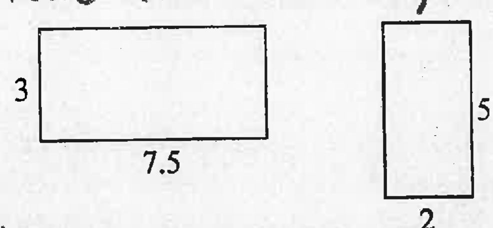


* All corresponding $\angle$s are $\cong$

$\frac{JM}{WZ} = \frac{4}{6} = \frac{2}{3}$ $\qquad \frac{JK}{WX} = \frac{4}{9}$ $\qquad \frac{ML}{ZY} = \frac{6}{12} = \frac{1}{2}$

* Corresponding sides are not ~ sides are not in proporti

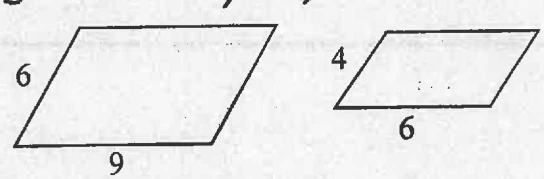
Ex: a. Are the two rectangles shown similar?



* All corresponding $\angle$s are $\cong$ because they are all right $\angle$s

$\frac{3}{2}$ $\qquad \frac{7.5}{5} = \frac{3}{2}$ $\qquad$ * Yes they are similar corresponding sides are in propor tion

b. Are the two parallelograms shown similar?



$\frac{6}{4} = \frac{3}{2}$ $\qquad \frac{9}{6} = \frac{3}{2}$

* Sides are in proportion

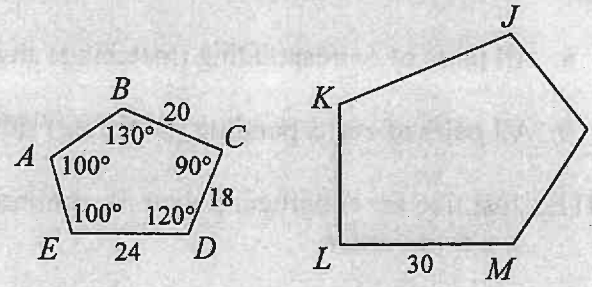
But we know nothing about the angles so can not tell if similar

Ex: In the diagram, $ABCDE \sim JKLMN$

a. Find $m\angle K$.

$\angle K \cong \angle B$ $\quad m\angle K = 130°$

b. Find $\overline{MN}$.



Big
Little

$$\frac{x}{24} = \frac{30}{18}$$

$$\frac{18x}{18} = \frac{720}{18}$$

$$x = 40$$

$$\boxed{MN = 40}$$

Geometry Notes S - 2: Proving Triangles Similar

Review: Two figures are similar if *there is a sequence of transformations that will map one onto the other*

In similar figures,

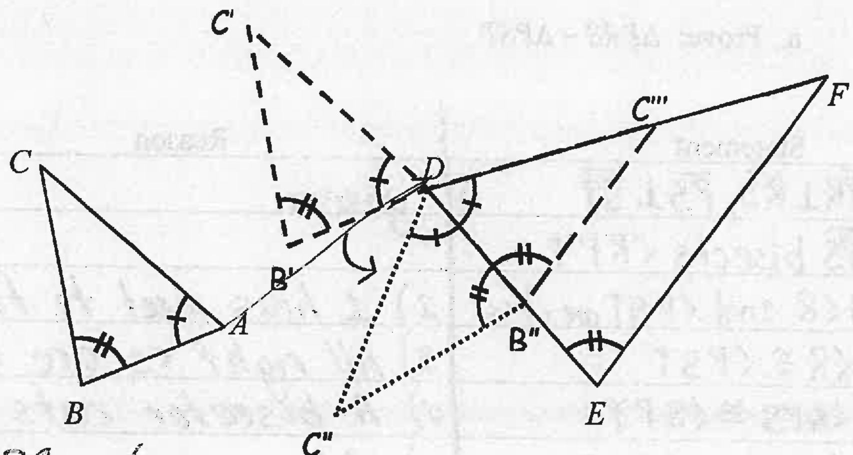
1. All pairs of corresponding angles are $\cong$
2. All pairs of corresponding sides are *in proportion*

Similar Triangles

Theorem: If two angles of one triangle are congruent to two angles of a second triangle, then the triangles are similar.

Given: ΔABC and ΔDEF
$\angle A \cong \angle D$, $\angle B \cong \angle E$

Show via a similarity transformation that ~~$\Delta A'B'C'$~~ $\sim \Delta ABC$.
ΔDEF



1) Translate ΔABC along vector $\overrightarrow{AD}$

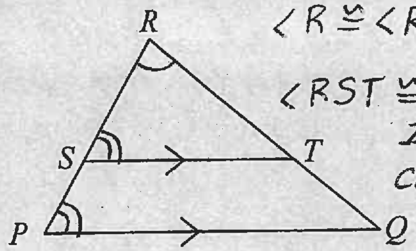
2) Rotate $\Delta DB'C'$ CCW around D $90°$

3) Reflect $\Delta DB''C''$ over DB''

4) Dilate $\Delta DB''C''$ by $r = \frac{EF}{BC}$

Ex: Write a similarity statement and give a reason why the triangles are similar.

a.

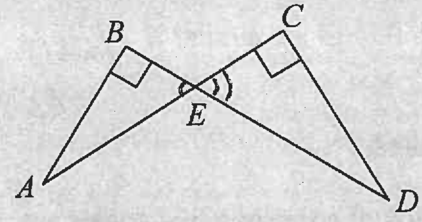


$\angle R \cong \angle R$ Reflexive

$\angle RST \cong \angle RPQ$ If lines are ∥ corresponding angles are $\cong$

$\triangle SRT \sim \triangle PRQ$ AA

b.

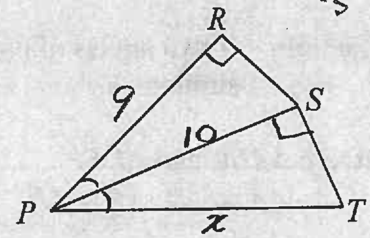


$\triangle ABE \sim \triangle DCE$

$\angle B \cong \angle C$ All Right $\angle$s $\cong$

$\angle BEA \cong \angle CED$ vertical $\angle$s $\cong$

Ex: Given: $\overline{PR} \perp \overline{RS}$, $\overline{PS} \perp \overline{ST}$, $\overline{PS}$ bisects $\angle RPT$.



a. Prove: $\triangle PRS \sim \triangle PST$.

Statement	Reason
1) $\overline{PR} \perp \overline{RS}$, $\overline{PS} \perp \overline{ST}$, $\overline{PS}$ bisects $\angle RPT$	1) given
2) $\angle R$ and $\angle PST$ are rt $\angle$s	2) $\perp$ lines meet to form right $\angle$s
3) $\angle R \cong \angle PST$	3) All right $\angle$s are $\cong$
4) $\angle RPS \cong \angle SPT$	4) A bisector cuts an $\angle$ into 2 $\cong$ $\angle$s
5) $\triangle PRS \sim \triangle PST$	5) AA

b. If $PR = 9$ and $PS = 10$, find PT.

$$\frac{x}{10} = \frac{10}{9}$$

$$\frac{100}{9} = \frac{9x}{9}$$

$$x = 11.1$$

$$\boxed{PT = 11.1}$$

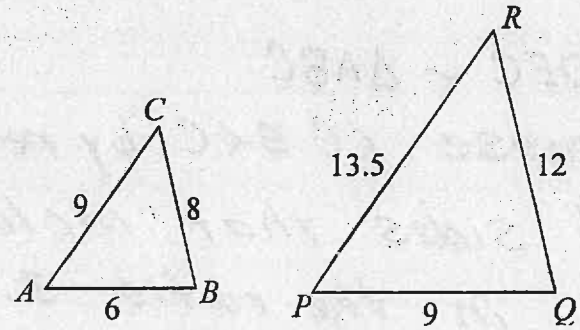
Geometry Notes S - 3: Proving Triangles Similar II

Theorem: If all three corresponding pairs of sides of two triangles are in proportion, then the triangles are similar. (Corresponding angles will automatically be congruent.)

Ex: Show the triangles at right are similar. SSS

$\frac{9}{6} = \frac{3}{2}$ $\frac{12}{8} = \frac{3}{2}$ $\frac{13.5}{9} = \frac{3}{2}$

All corresponding sides are in proportion

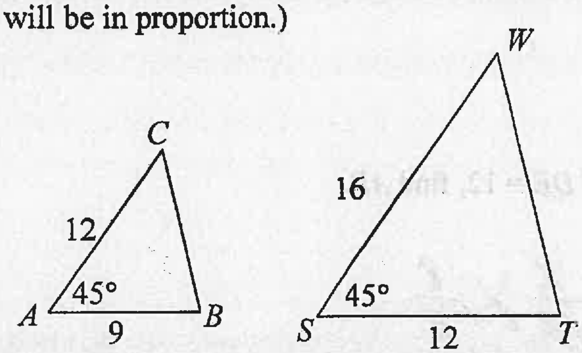


Theorem: If one pair of corresponding angles of two triangles is congruent and the sides that include those angles are in proportion, then the triangles are similar. (The other two pairs of corresponding angles will be congruent and the third pair of sides will be in proportion.)

Ex: Show the triangles at right are similar. SAS

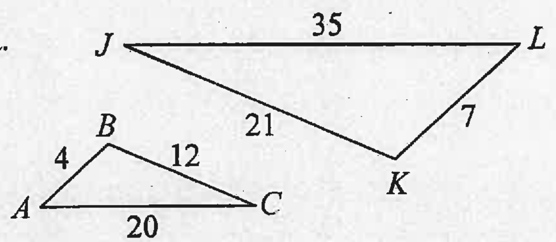
$\frac{16}{12} = \frac{4}{3}$ $\frac{12}{9} = \frac{4}{3}$

* One pair of corresponding ∠s are ≅
* Sides that include the ∠s are in proportion



Ex: Determine if the two triangles are similar.

a.



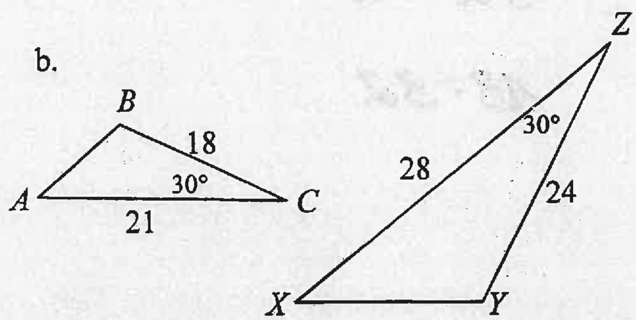
$\frac{35}{20} = \frac{7}{4}$

$\frac{7}{4}$

$\frac{21}{12} = \frac{7}{4}$

* Yes similar all corresponding sides are in proportion

b.

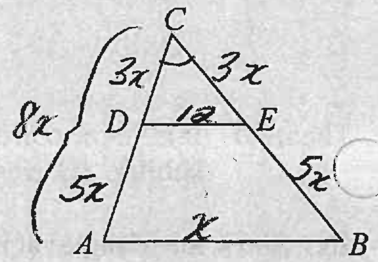


$\frac{28}{21} = \frac{4}{3}$

$\frac{24}{18} = \frac{4}{3}$

* Yes similar one pair of corresponding ∠s are ≅ sides that include the ∠s are in proportio

Ex: Given: ΔABC, D divides $\overline{CA}$ in the ratio 3:5 and E divides $\overline{CB}$ in the ratio 3:5.



a. Prove: $\Delta DEC \sim \Delta ABC$.

$\Delta DEC \sim \Delta ABC$

because $\angle C \cong \angle C$ by reflexive and sides that include $\angle C$ are in the ratio 3:8

b. If $DE = 12$, find AB.

$$\frac{x}{12} = \frac{8}{3}$$

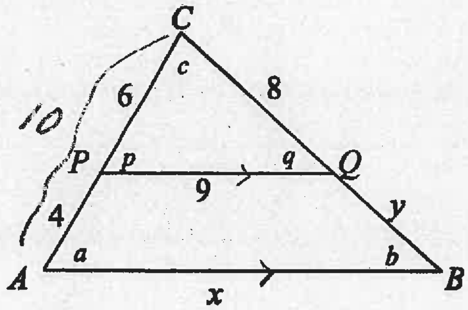
$$\frac{96}{3} = \frac{3x}{3}$$

$$32 = x$$

$$AB = 32$$

Geometry Notes S - 4: Solving Similar Triangle Problems

Ex: In the diagram, $\overline{PQ} \parallel \overline{AB}$.



a. Is $\Delta ABC \sim \Delta PQC$?

Yes, $\angle C \cong \angle C$ reflexive

since lines are // corresponding

$\angle$s are $\cong$ ... $\angle p \cong \angle a$, $\angle q \cong \angle b$

* since all corresponding $\angle$s are $\cong$ Δs are similar

b. Find the values of x and y.

$\frac{6}{8} \times \frac{4}{y}$

$\frac{\cancel{6}y}{\cancel{6}} = \frac{32}{6}$

$y = 5.3$

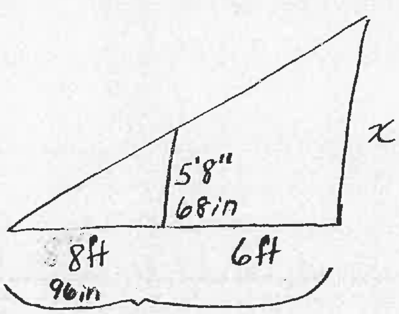
$\frac{x}{9} \times \frac{10}{6}$ ← must have whole side

$\frac{90}{6} = \frac{\cancel{6}x}{\cancel{6}}$

$15 = x$

*when lines are // you can set up sides across from each other

Ex: Amber stands up on stage trying out for a role in a play. A footlight shines on her from eight feet in front of her. Six feet behind her is a tall light colored backdrop. If Amber is five feet eight inches tall, how tall is her shadow on the backdrop?



14ft

168in

$\frac{168}{96} \times \frac{x}{68}$

$\frac{11424}{96} = \frac{\cancel{96}x}{\cancel{96}}$

$\boxed{119in = x}$

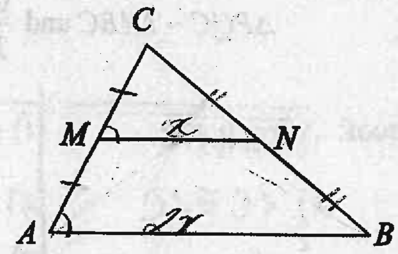
Geometry Notes S - 6: Midpoints and Parallel Lines

Theorem: If a segment joins the midpoints of two sides of a triangle then it is parallel to and half the length of the third side of the triangle.

Given: ΔABC, M is the midpoint of $\overline{AC}$ and N is the midpoint of $\overline{BC}$.

Prove: a. $\frac{MN}{AB} = \frac{1}{2}$

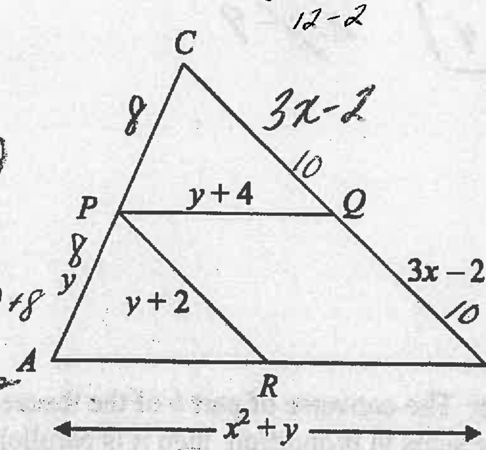
b. $\overline{MN} \parallel \overline{AB}$



Statement	Reason
1) M midpt $\overline{AC}$ N midpt $\overline{BC}$	1) given
2) $\overline{CM} \cong \overline{MA}$ $\overline{CN} \cong \overline{NB}$	2) A midpt cuts a segment into 2 ≅ segments
3) $\angle C \cong \angle C$	3) Reflexive
4) $\Delta MCN \sim \Delta ACB$	4) SAS
5) $\frac{MN}{AB} = \frac{1}{2}$	5) In ~ Δs corresponding sides are in proportion
6) $\angle CMN \cong \angle CAB$	6) Corresponding ∠s in ~ Δs are ≅
7) $\overline{MN} \parallel \overline{AB}$	7) If corresponding ∠s are ≅ lines are ∥

Ex: In the diagram, P, Q and R are the midpoints of the sides of ΔABC. Find the perimeter of ΔABC.

$3(4)-2$
$12-2$



4^2+8
$16+8$
24

$2(3x-2) = 2(y+2)$

$6x-4 = 2y+4$
$-4 \quad -4$

$\frac{6x-8}{2} = \frac{2y}{2}$

$3x-4 = y$

$3(4)-4 = y$

$12-4 = y$

$\boxed{8 = y}$

$x^2+y = 2(y+4)$

$x^2+y = 2y+8$

$x^2+3x-4 = 2(3x-4)+8$

$x^2+3x-4 = 6x-\cancel{8}+\cancel{8}$
$-6x \quad -6x$

$x^2-3x-4 = 0$

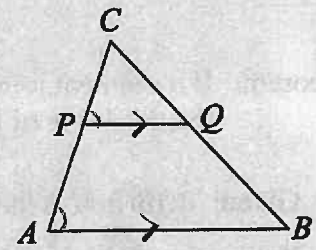
$(x+1)(x-4)$

$\cancel{x=-1}$ $\boxed{x=4}$

$P = 8+8+10+10+24$

$\boxed{P = 60}$

Theorem: If a line parallel to one side of a triangle intersects the other two sides, then
a. it forms two similar triangles and
b. it divides the intersected sides in proportion.



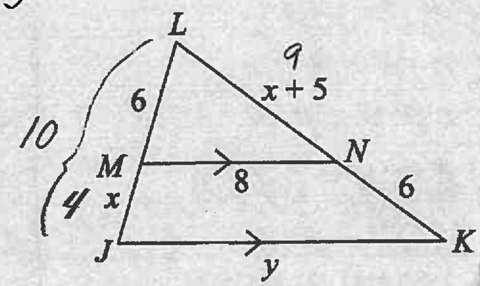
If $\overline{PQ} \parallel \overline{AB}$, then

$\Delta PQC \sim \Delta ABC$ and $\frac{CP}{PA} = \frac{CQ}{QB}$

Proof:

1) $\overline{PQ} \parallel \overline{AB}$	1) given
2) $\angle C \cong \angle C$	2) Reflexive
3) $\angle CPQ \cong \angle CAB$	3) If lines are ∥ corresponding ∠s ≅
4) $\Delta PQC \sim \Delta ABC$	4) AA
* 5) $\frac{CP}{PA} = \frac{CQ}{QB}$	5) ∥ lines divide intersecting sides in proportion

Ex: Solve for x and y in the diagram at right.



$\frac{6}{x+5} = \frac{x}{6}$

$x(x+5) = 36$

$x^2 + 5x - 36 = 0$

$(x-4)(x+9) = 0$

$\boxed{x = 4}$ $\quad x \neq -9$

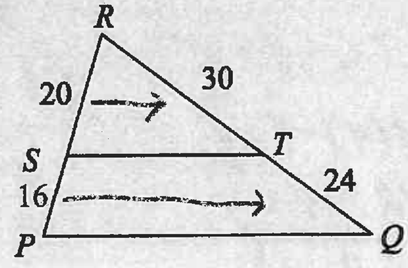
$\frac{y}{8} = \frac{10}{6}$

$\frac{80}{6} = \frac{6y}{6}$

$\boxed{13.3 = y}$

Note: The converse of part *b* of the theorem is also true: If a line intersects two sides of a triangle and divides those sides in proportion, then it is parallel to the third side.

Ex: Is $\overline{PQ} \parallel \overline{ST}$?



$\frac{20}{30} = \frac{2}{3} \qquad \frac{16}{24} = \frac{2}{3}$

Yes

Geometry Notes S - 7: Perimeters, Areas and Volumes of Similar Figures

Review: Lengths are measured in in ft

Areas are measured in in^2 ft^2

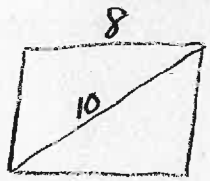
Volumes are measured in in^3 ft^3

Facts: If two figures are similar,

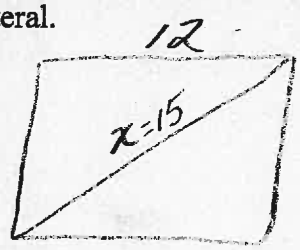
1. All pairs of corresponding segments (lengths) are in the same ratio.
   This includes: sides, medians, altitudes, angle bisectors, diagonals, perimeters
2. The ratio of areas is the $\boxed{\text{square}}$ of the ratio of the sides.
3. The ratio of volumes is the $\boxed{\text{cube}}$ of the ratio of sides.

Ex: The lengths of the longest sides of two similar quadrilaterals are 8 and 12.

a. If a diagonal of the smaller quadrilateral measures 10, find the length of the corresponding diagonal in the second quadrilateral.



$A = 60$



$$\frac{12}{8} = \frac{x}{10}$$

$$120 = 8x$$

$$x = 15$$

b. if the area of the smaller quadrilateral is 60, find the area of the larger quadrilateral.

$$r = \frac{12}{8} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$A = 60 \times \frac{9}{4} = \frac{540}{4} = \boxed{135}$$

Ex: Joe A. Guy is 6 feet tall. He weighs 175 pounds and the total volume of his body is $\boxed{\text{2.8 cubic feet}}$. In a science experiment gone horribly wrong, Joe manages to enlarge himself by a factor of 8; he is now 48 feet tall.

a. What is Joe's new volume?

$$r = \frac{new}{old} = \frac{48}{6} = 8 \qquad V = 2.8 \times 512$$

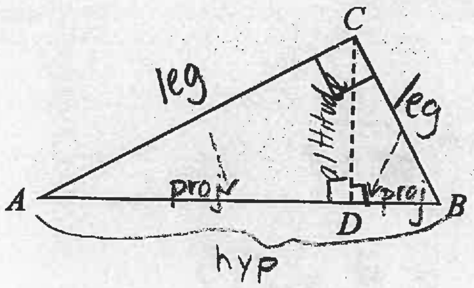
$$= 8^3 = 512 \qquad = \boxed{1433.6 ft^3}$$

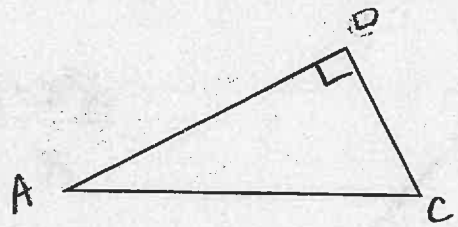
b. What is Joe's new weight?

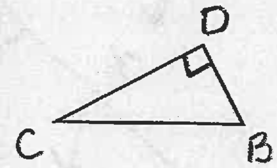
$$175 \times 8 = 1400 lbs$$

Geometry Notes S - 9: Similarity in Right Triangles

Theorem: The altitude to the hypotenuse of a right triangle forms three similar triangles.







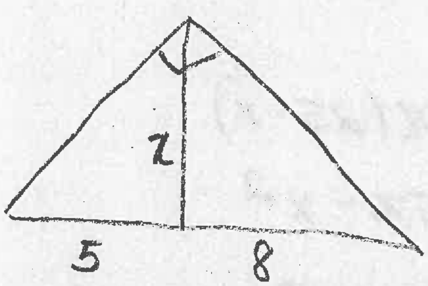
Leg Rule

$$\frac{\text{Whole Hyp}}{\text{leg}} = \frac{\text{leg}}{\text{projection}}$$

Altitude Rule

$$\frac{\text{Part of hyp}}{\text{Alt}} = \frac{\text{Alt}}{\text{other part}}$$

Ex: The altitude to the hypotenuse of a right triangle divides the hypotenuse into segments of length 5 and 8. What is the measure of the altitude?



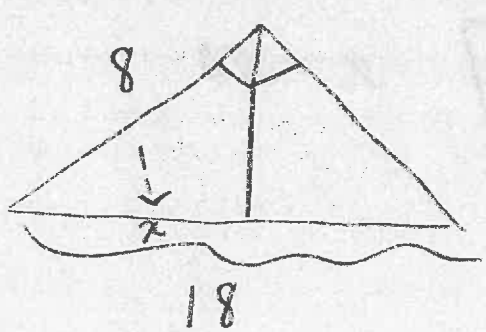
$$\frac{5}{x} = \frac{x}{8}$$

$$\sqrt{x^2} = \sqrt{40}$$

$$x = \sqrt{4}\sqrt{10}$$

$$\boxed{x = 2\sqrt{10}}$$

Ex: In a right triangle, the hypotenuse measures 18 and one leg measures 8. Find the length of the projection of that leg on the hypotenuse.

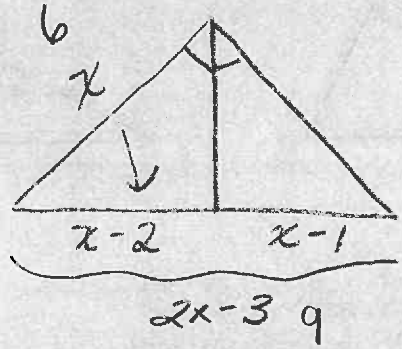


$$\frac{18}{8} = \frac{8}{x}$$

$$\frac{64}{18} = \frac{18x}{18}$$

$$\boxed{x = 3.56}$$

Ex: In a right triangle, the length of the projection of the shorter leg on the hypotenuse is two less than the length of the shorter leg. The length of the projection of the longer leg on the hypotenuse is one less than the length of the shorter leg. Find the lengths of all three sides of the triangle.



$$\frac{2x-3}{x} = \frac{x}{x-2}$$

$$x^2 = 2x^2 - 7x + 6$$

$$0 = x^2 - 7x + 6$$

$$0 = (x-1)(x-6)$$

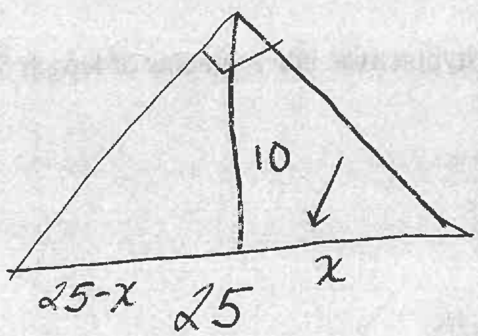
$$\cancel{x=1} \quad x=6$$

	$2x$	-3
x	$2x^2$	$-3x$
-2	$-4x$	6

$$6^2 + b^2 = 9^2$$

$$36 + b^2 = 81 \quad \sqrt{b^2} = \sqrt{45} \quad b = \sqrt{45} = 6.7$$

Ex: In a right triangle, the hypotenuse measures 25 and the altitude to the hypotenuse measures 10. Find the length of the projection of the shorter leg on the hypotenuse.



$$\frac{25-x}{10} = \frac{10}{x}$$

$$100 = x(25-x)$$

$$100 = 25x - x^2$$

$$x^2 - 25 + 100 = 0$$

$$(x-5)(x-20) = 0$$

$$\boxed{x=5} \quad x = 20$$