# BASIC CONSTRUCTIONS

**Notes:**
1. All segments are *drawn* with a straight-edge.
2. All segments are *measured* with a compass (*not* a ruler!).
3. The various arcs and segments created are called "construction marks."  Do *not* erase them when you're done.  If they do not all appear on your paper, you get *no credit* for the problem!

**There are seven basic constructions:**

**1.  Construct a segment congruent to a given segment $\overline{AB}$.**
   1. Locate a point $A'$ to be one endpoint of the new segment. (Sometimes the point is given.)
   2. Measure segment $\overline{AB}$.  Keeping compass the same size, move compass point to $A'$ and make arc in the area where you want the new segment to end.  (Sometimes a ray is given.)
   3. Draw a segment from $A'$ to any point on the arc, $B'$.
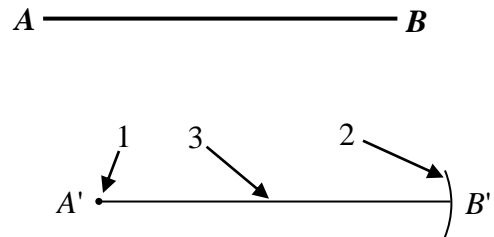
**2.  Construct the perpendicular bisector of a given segment $\overline{AB}$.**
   1. Open compass to width greater than $\frac{1}{2}(AB)$.
   2. With compass point on $A$, make large arc crossing $\overline{AB}$ twice. (Or two small arcs, one above, one below $\overline{AB}$.)
      Keeping compass the same size, repeat with compass point on $B$.
   3. With straight-edge, connect intersections of arcs.
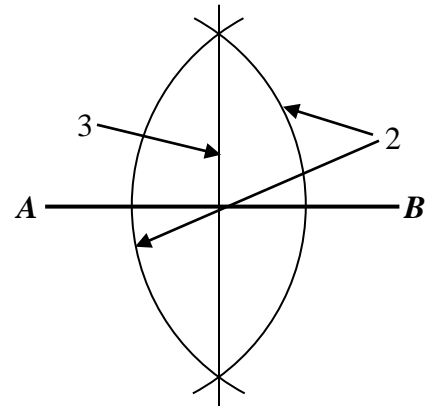
**3.  Construct an angle congruent to a given angle $ABC$.**
   1. Draw ray $\overrightarrow{B'C'}$ (it is not necessary that it be congruent to $\overline{BC}$).
   2. With compass point on $B$, draw arc over both sides of $\angle ABC$.  (The points where the arc intersects the sides of the angle have been labeled $P$ and $Q$ for convenience.)
   3. Without changing compass size, move point to $B'$ and draw similar arc.
   4. Measure $\overline{PQ}$.  Keeping compass the same size, place compass point on $P'$ and draw arc intersecting original arc.  This intersection is $Q'$.
   5. Draw ray $\overrightarrow{BQ'}$.
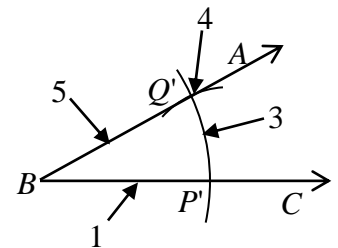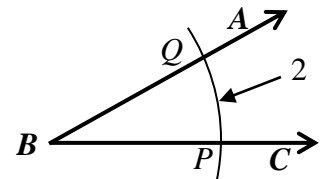
**4.  Bisect a given angle $ABC$.**
   1. With point on vertex $B$, draw an arc through both sides of $\angle ABC$. (The points where the arc intersects the sides of the angle have been labeled $P$ and $Q$ only for convenience.)
   2. With point on $P$, draw arc shown in diagram.  Without changing compass size, repeat with point at $Q$.
   3. Draw bisector from vertex out through intersection of arcs from 2.
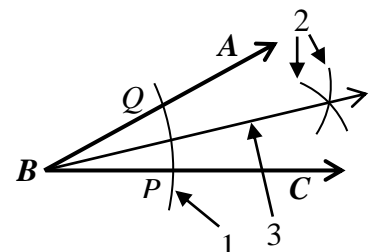
**5. Construct a perpendicular to a given line *l* through a given point *P* not on *l*.**
1. With point on *P* and compass opened wider than distance *P* to *l*, draw two arcs on *l* (or one big arc intersecting *l* twice).
2. With point on intersection of first arc and *l*, draw new arc below *l*. Keeping compass the same size, repeat with point on intersection of second arc and *l*.
3. Draw segment from *P* through intersection of the newest two arcs.
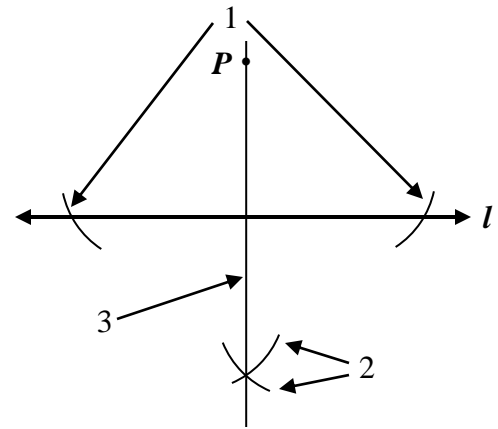
**6. Construct a perpendicular to a given line *l* through a given point *P* on *l*.**
1. With point on *P*, make two arcs on *l*, one each side of *P*.
2. Open compass wider. With point on intersection of one arc and *l*, make an arc above *P*. Keeping compass the same size, repeat with point on intersection of second arc and *l*.
3. Draw segment from intersection of newest two arcs through *P*.
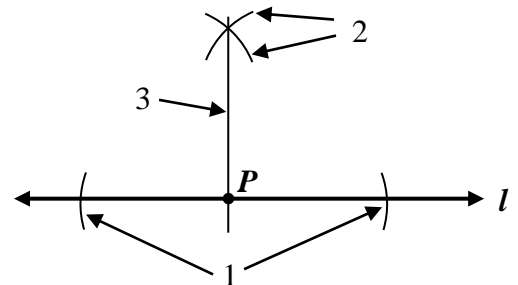
**7. Construct a line parallel to a given line *l* through a given point *P* not on *l*.**
1. Draw any convenient line through *P* and intersecting *l*.
2. Follow directions for constructing congruent angles to construct ∠*b* congruent to ∠*a*.
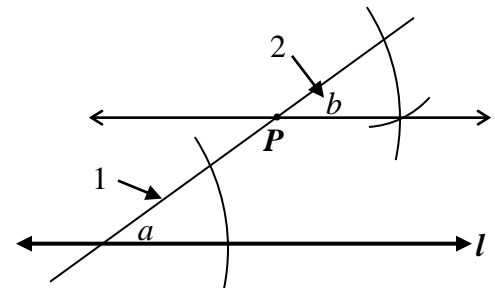
Alternative method:
1. Locate a convenient point on *l*; label it *A*.
2. With center at *A* and radius *AP*, draw an arc that intersects line *l*, label the intersection *B*.
3. Keeping radius *AP*, make arcs centered at *P* and *B* and intersecting on the opposite side of arc *PB* from *A*. Label the intersection *Q*.
4. Draw $\overline{PQ}$.
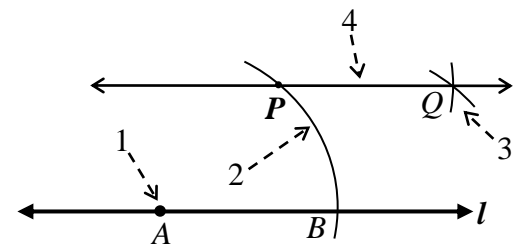
**8. Construct the line of reflection given a figure and its image.**
1. Draw the segment connecting any point on $\triangle ABC$ to its image of $\triangle A'B'C'$.
2. Construct the perpendicular bisector of that segment.
   b. Construct the perpendicular bisector of that segment.
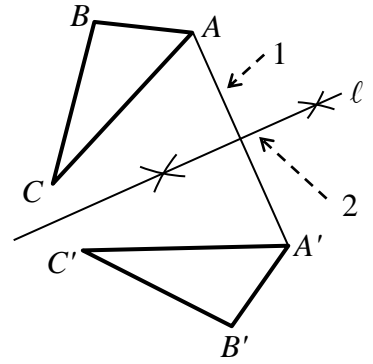
**9. Locate the center of rotation and show the angle of rotation given a figure and its image.**
1. Draw $\overline{AA'}$ and $\overline{BB'}$. (Any two points and their images will do as long as the segments are not parallel.)
2. Construct the perpendicular bisectors of $\overline{AA'}$ and $\overline{BB'}$. Label the point where they intersect $O$. This is the center of rotation.
3. Draw $\overline{OA}$ and $\overline{OA'}$. $\angle AOA'$ is the angle of rotation.
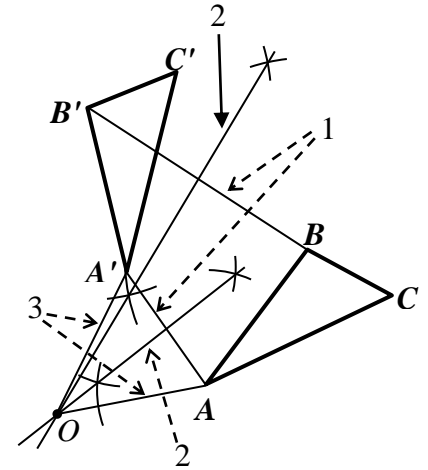
**10a. Construct a diameter of a circle.**
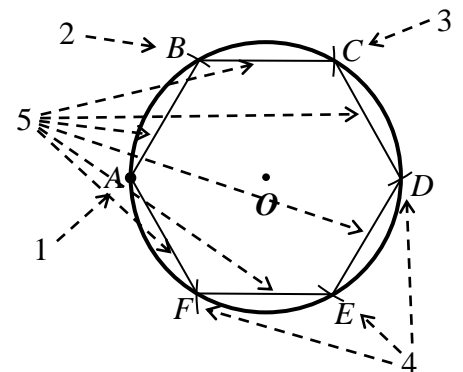   **b. Locate the center of a circle.**
1. Draw any convenient chord $\overline{AB}$.
2. Construct the perpendicular bisector of $\overline{AB}$; label the points where it intersects the circle $C$ and. $\overline{CD}$ is a diameter.
3. (Not shown) Construct the midpoint (perpendicular bisector of $\overline{CD}$. This point is the center of the circle.
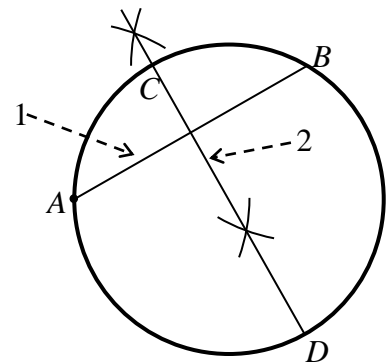
**11a. Construct a regular hexagon inscribed in a circle.**
   **b. Construct an equilateral triangle inscribed in a circle.**
   Note: If the center of the circle is not given, it must be found first.
1. Locate any point on the circle and label it $A$.
2. With center at $A$ and radius $AO$, construct an arc that intersects the circle; label the intersection $B$.
3. With center at $B$ and radius $AO$, construct an arc that intersects the circle on the other side of $B$ from $A$; label the intersection $C$.
4. Continue as above until you have $D$, $E,$ and $F$ on the circle.
5. Draw $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EF}$, and $\overline{FA}$.

   (Not shown) For an equilateral triangle, skip every other point in step 5: draw $\overline{AC}$, $\overline{CE}$, and $\overline{EA}$ or $\overline{BD}$, $\overline{DF}$, and $\overline{FB}$.

**12. Construct a square inscribed in a circle.**

Note: If the center of the circle is not given, it must be found first.

1. Locate any point on the circle and label it *A*.

2. Draw radius $\overline{AO}$; extend it to become diameter $\overline{AOC}$.

3. Construct the perpendicular bisector of $\overline{AOC}$; label the points where it intersects the circle *C* and *D*.

4. Draw $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{DA}$.
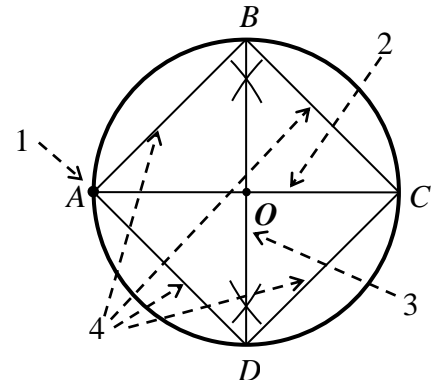
**13. Construct the circumcircle for a triangle.**

1. Constrict the perpendicular bisectors of any two sides. Label their point of intersection *O*.

2. With center *O* and radius *OA* (or *OB* or *OC*), draw the circumscribed circle.
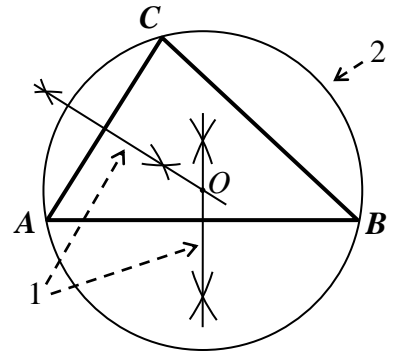
**14. Construct the incircle for a triangle.**

1. Bisect any two angles. Label the intersection of the bisectors *O*.

2. Construct a line through *O* perpendicular to any side of $\triangle ABC$. Label the point where it intersects the side *R*.

3. Draw the incircle with center at *O* and radius *OR*.