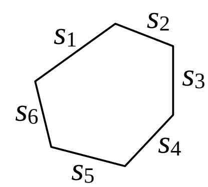
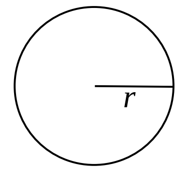
Geometry Notes PAV - 4: Perimeter and Area

<u>Perimeter</u>

Polygon:

Circle:

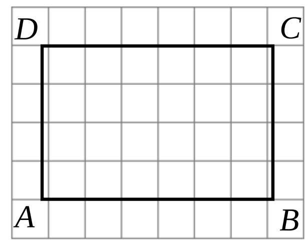




<u>Area</u>

Rectangle

Ex: In the diagram at right, each space represents one foot. Find



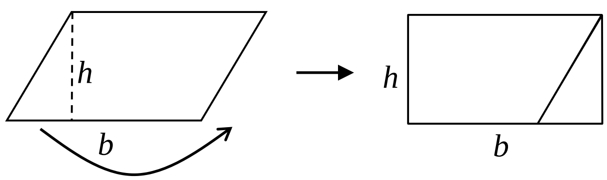
a. The length of $\overline{AB}$

b. The length of $\overline{AD}$

c. The area of *ABCD*.

The area of a rectangle is

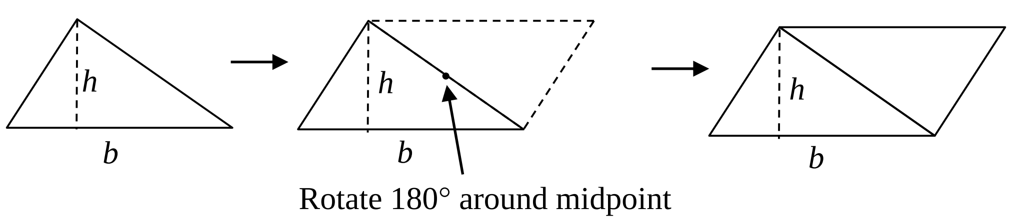
Parallelogram



A parallelogram has the same area as a rectangle with the same base and height.

The area of a parallelogram is

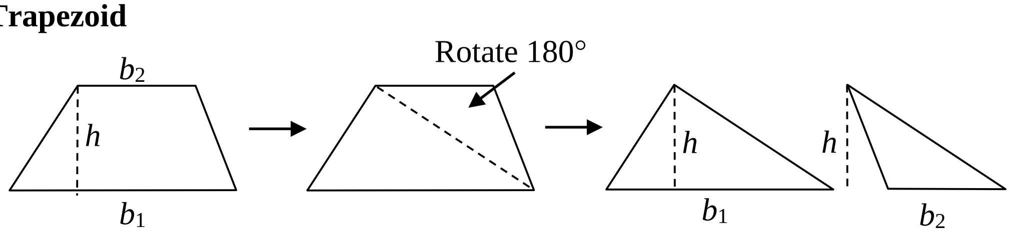
Triangle



A triangle has half the area of a parallelogram with the same base and height.

The area of a triangle is

Trapezoid



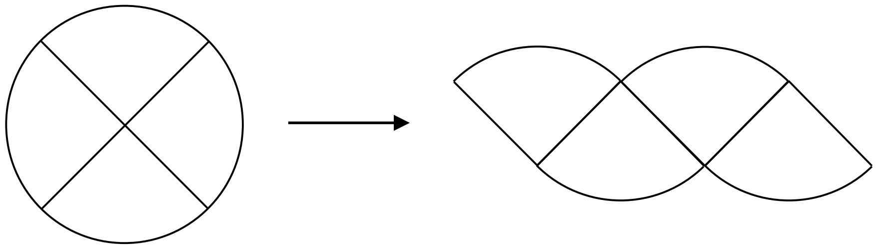
A trapezoid has area of the sum of two triangles with the same height.

The area of a trapezoid is

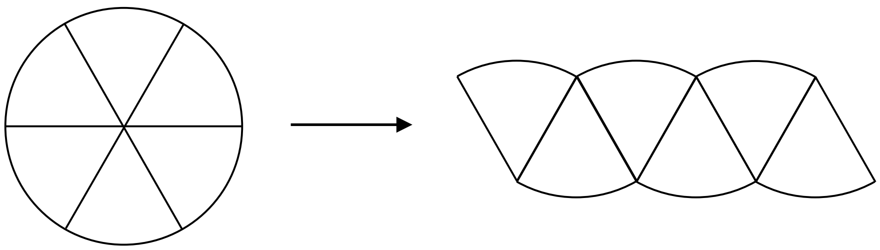
Area of a Circle

Divide the circle into n equal sectors, then rearrange.

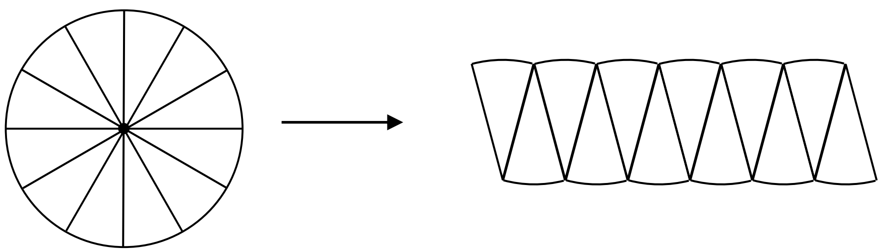
$n = 4$



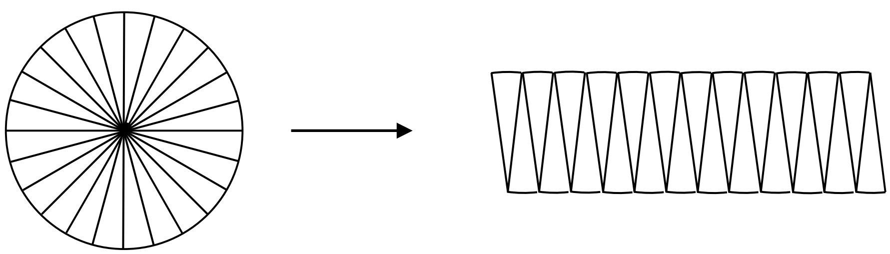
$n = 6$



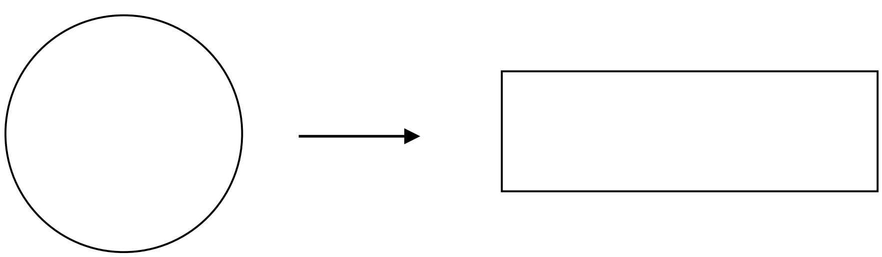
$n = 12$



$n = 24$



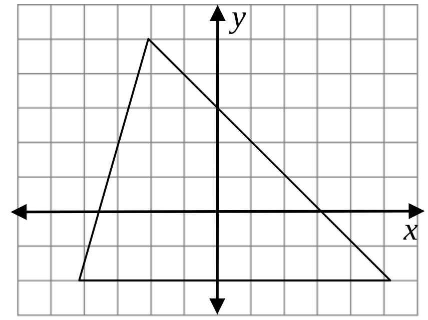
$n \to \infty$



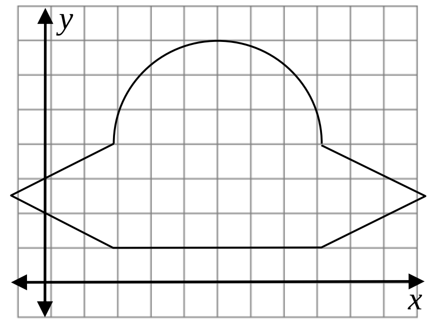
The area of a circle is

Areas in Coordinate Geometry

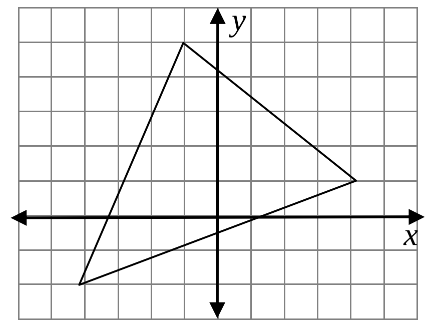
Ex: Find the area of the triangle.



Ex: Find the area of the figure which consists of five line segments and a semicircle.



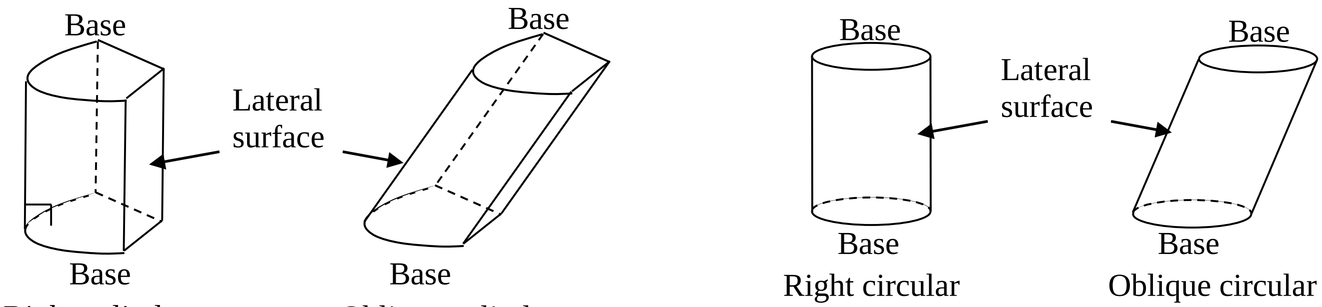
Ex: Find the area of the triangle.



Geometry Notes PAV - 5: Cylinders & Prisms, Volume

<u>Cylinders</u>

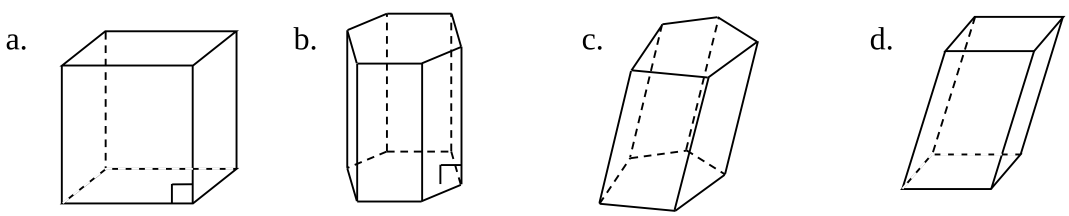
A *cylinder* has two *bases* that are congruent, parallel closed figures with *lateral surfaces* connecting them.
- In a *right cylinder*, the lateral surfaces are perpendicular to the bases
- In an *oblique cylinder*, the lateral sides are not perpendicular to the bases
- In common usage, *cylinder* almost always means *circular* cylinder but the bases may be other shapes



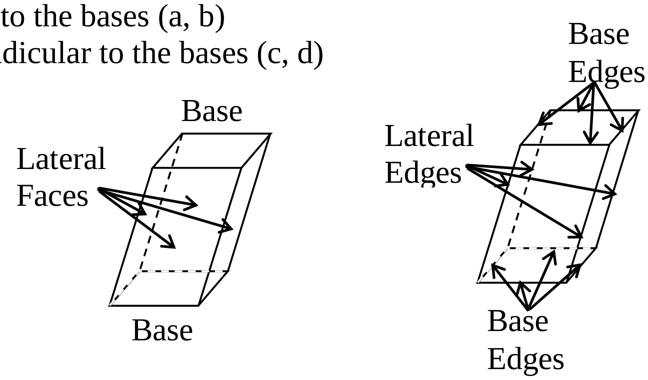
Right cylinder   Oblique cylinder   Right circular cylinder   Oblique circular cylinder

<u>Prisms</u>

A *prism* is a cylinder where the bases are *polygons*.

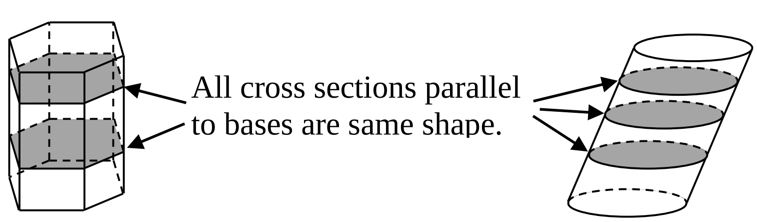


- Prisms are named by the shape of their base:
  e.g., rectangular prism (a), hexagonal prism (b), pentagonal (c), square (d)
- In a *right prism*, the lateral sides are perpendicular to the bases (a, b)
- In an *oblique prism*, the lateral sides are not perpendicular to the bases (c, d)
- The bases and lateral sides are called *faces*.
  -- The bases can be any polygon.
  -- The lateral faces will be parallelograms
  (rectangles in case of a right prism).
- The intersections of faces are *edges*
  -- Edges at bases are *base edges*
  -- Edges between lateral faces are *lateral edges*
  Lateral edges are all *parallel* and *congruent*
- Where edges intersect are *vertices*



Important property of prisms and cylinders:

All *cross sections* ("slices") parallel to the bases will have the same shape, the shape of the base.

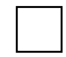


Volume

Remember: Lengths are measured in *units* of length (inches, centimeters, feet, meters, etc)

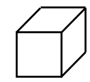
One unit

Areas are measured in *square units* (square inches, cm^2, ft^2, m^2, etc)



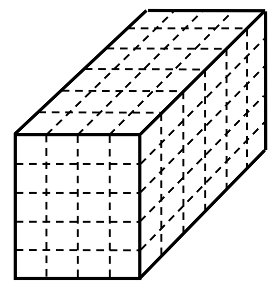
One square unit

Volumes are measured in *cubic units* (cubic inches, cm^3, ft^3, m^3, etc)

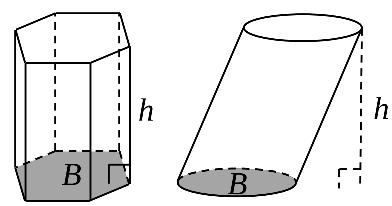


One cubic unit

Ex: Find the volume of the rectangular prism at right.

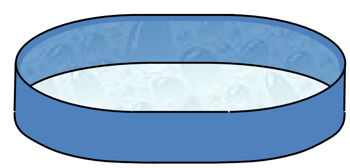


Volume of a prism or cylinder



It does not matter
- what shape the base is
- whether its right or oblique

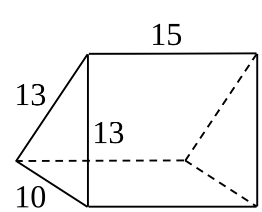
Ex: An above ground pool covers an oval area of 185 ft^2 and is filled to a depth of 4 ft. What is the volume of water in the pool?



If the density of water is about 62.3 pounds per cubic foot, what is the total weight of the water in the pool?

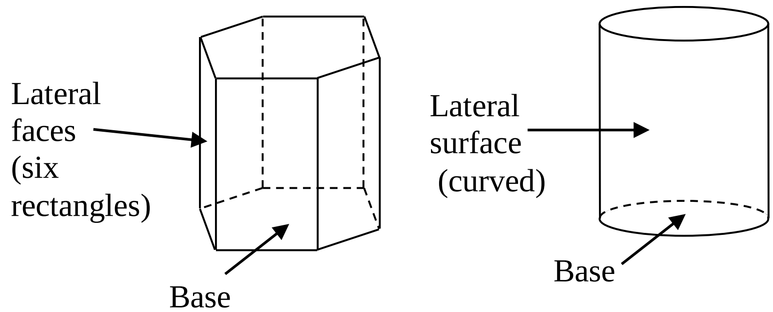
Ex: Find the volume of a right circular cylinder with a radius of 4 cm and a height of 12 cm.

Ex: Find the volume of the prism at right.



Surface Area

The *surface area* of a prism/cylinder has two parts:

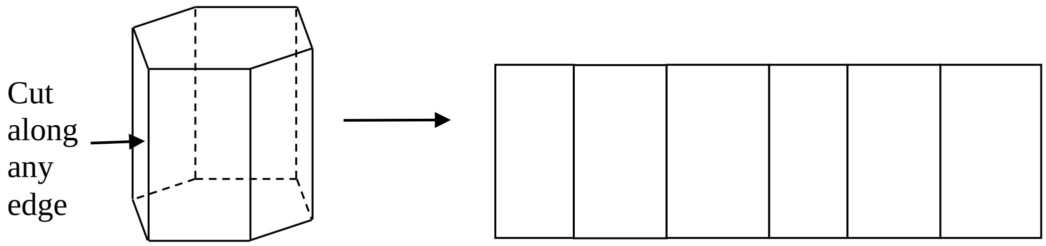


Lateral area: The area of the lateral faces or surface, and

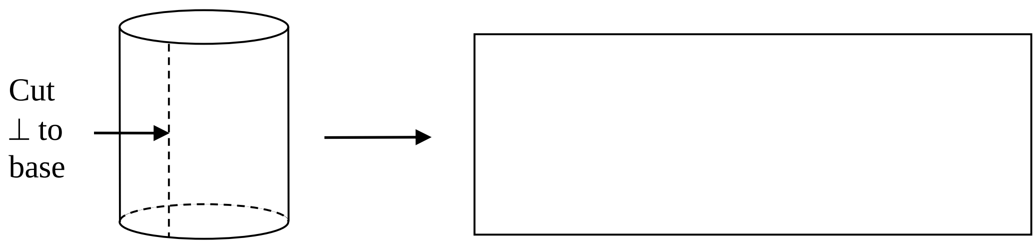
Base area: The area of the two bases

Lateral Area

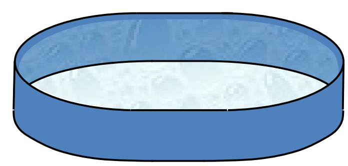
For *right* prism:



For *right* cylinder:

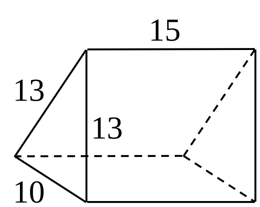


Ex: The pool from the earlier problem has a perimeter of 50 ft and a height of 4.5 ft. Find the total surface area of (one side of) the pool liner.



Ex: Find the surface area of a right circular cylinder with a radius of 4 cm and a height of 12 cm.

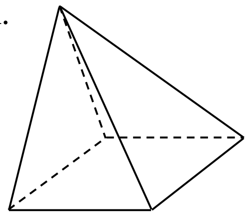
Ex: Find the surface area of the prism at right.

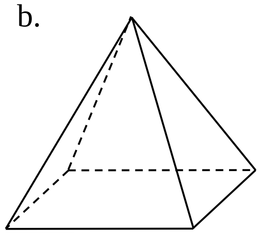


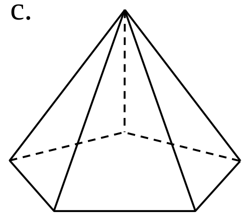
Geometry Notes PAV - 7: Cones and Pyramids

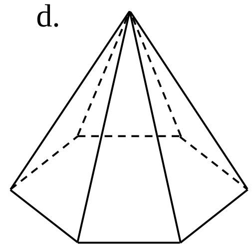
Pyramid

A *pyramid* has a single base that is a polygon and a single *apex*, a point not inn the plane of the base. Each vertex of the base is joined to the apex by a *lateral edge*.

a. 

b. 

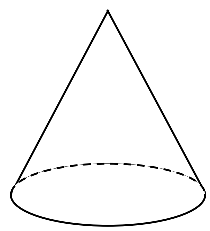
c. 

d. 

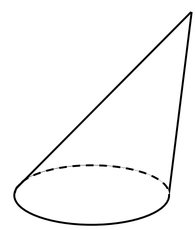
- Pyramids are named by the shape of their base
  - -- Square (a), rectangular (b), pentagonal (c), hexagonal (d)
- A *right pyramid* has its apex directly above the center of its base (b, c, d)
- An *oblique pyramid* does not have its apex directly above its base (a)
- A *regular pyramid* is a right pyramid whose base is a *regular polygon* (c, d)
  - -- In a regular pyramid, all the lateral faces are congruent isosceles triangles.

Cone

A *cone* is like a pyramid except that the base is a *curved figure*.
- In common usage, *cone* almost always means *circular cone* but the bases may be other shapes (such as ellipses)
- Like pyramids, cones may be *right* (most common) or *oblique*



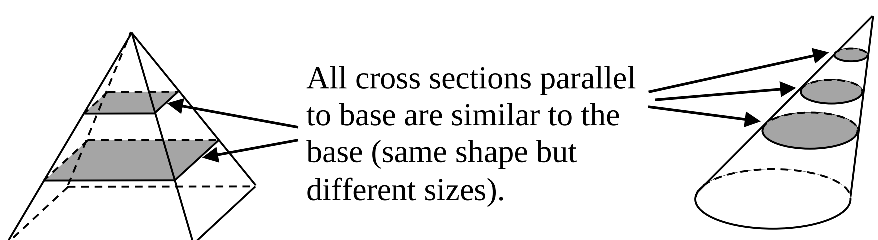
Right circular cone.



Oblique circular cone.

Important property of pyramids and cones:

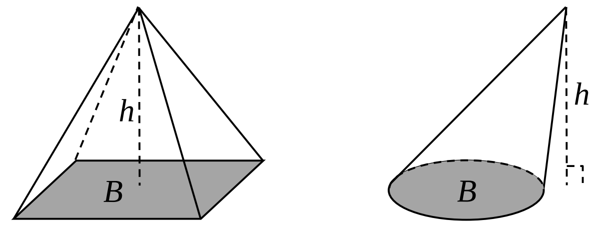
All *cross sections* ("slices") parallel to the base will be *similar* to the base.



Oblique circular cone.

Volumes of Pyramids and Cones

For *any* pyramid or cone, the volume is given by



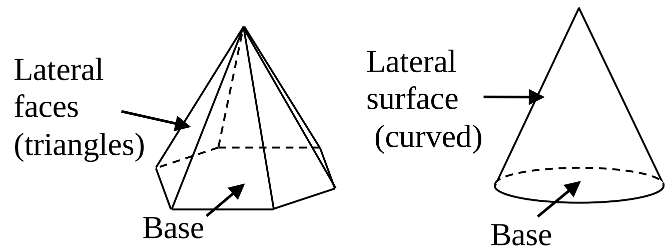
The volume of a pyramid/cone is $\frac{1}{3}$ the volume of a prism/cylinder with the same base and height.

Ex: The Pyramid of Menkaure has a square base of length 106 meters and a height of 61 meters. If the rock the pyramid is made of has an average density of 2200 kg/m^3, find he total mass of the pyramid.

Ex: The atrium of the National Museum of the Marine Corps is an oblique circular cone with a base diameter of 180 feet and a height (not counting the mast) of 135 feet. What is its volume?

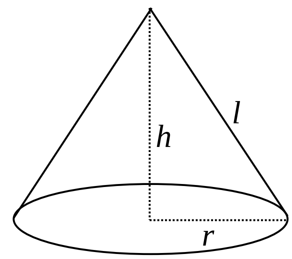
Surface Area

The total surface area of a pyramid/cone has of two parts:

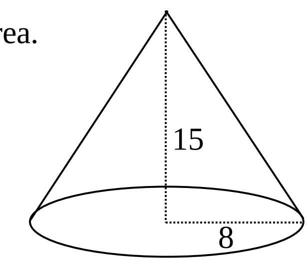


Base area: The area of the base
Lateral area: The area of the lateral faces or surface.

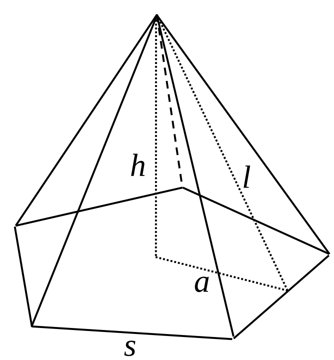
For a *right circular cone* the lateral surface area is



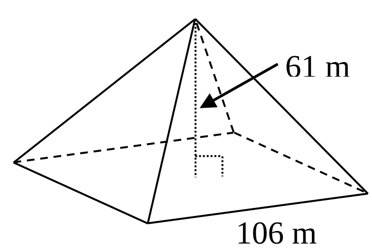
Ex: A right circular cone has a radius of 8 and a height of 15. Find its lateral surface area.



For a *regular pyramid*, the *lateral* surface area can be found by



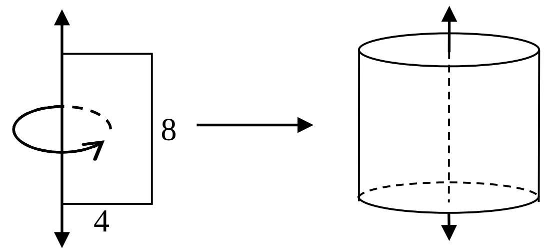
Ex: Find the lateral surface area of the Pyramid of Menkaure.



Geometry Notes PAV - 8: Volumes of Rotation, Cross-sections, Cavalieri's Principle

Volumes of Rotation

Ex: A 4 by 8 rectangle is rotated about the line containing one of its long sides.



a. What shape is the resulting solid?

b. What is the volume?

c. What is the total surface area?

Ex: A right triangle with legs $\sqrt{7}$ and 3 is rotated about the line containing the longer leg.

a. What shape is the resulting solid

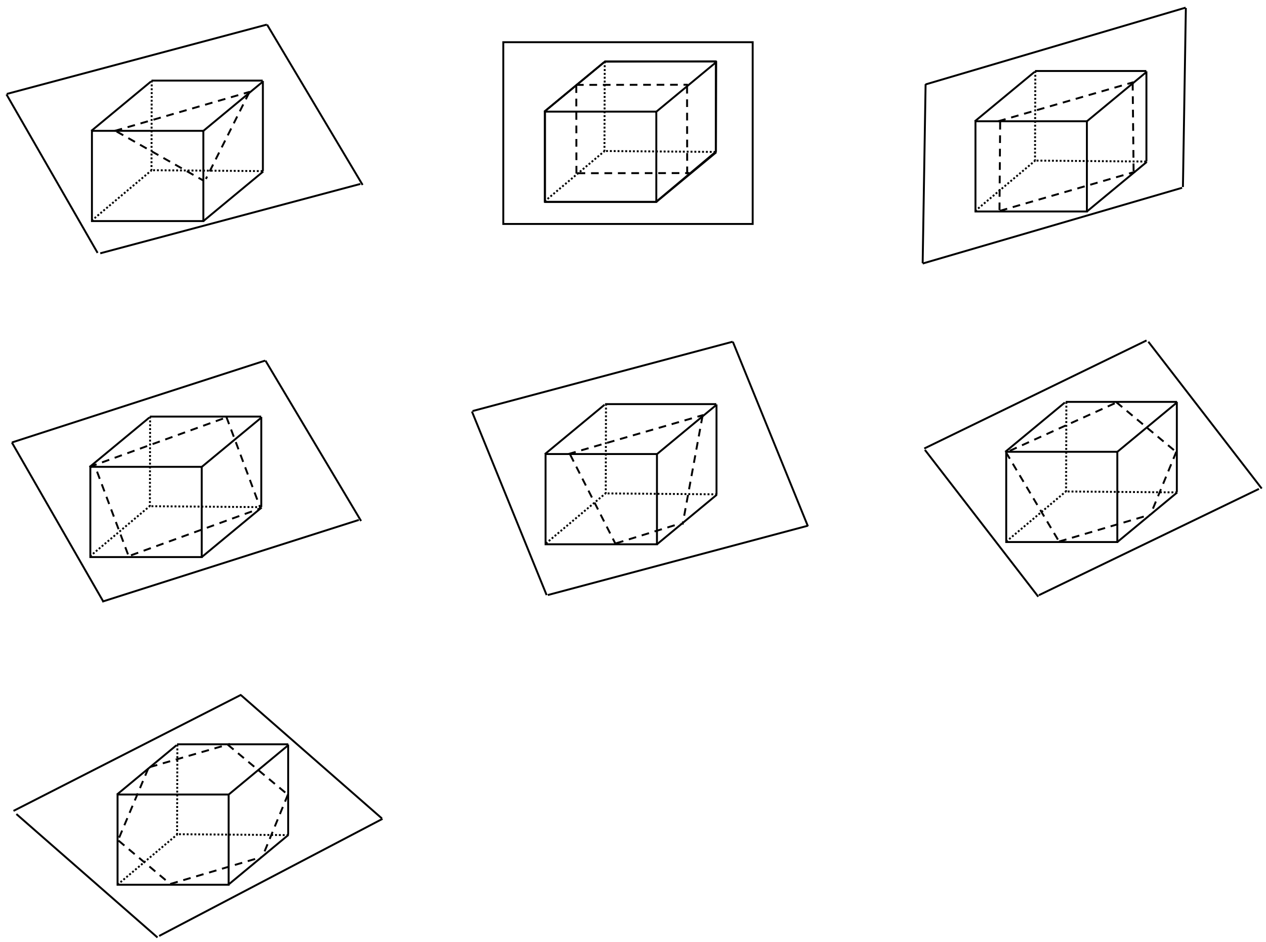
b. What is the volume?

c. What is the total surface area?

Cross-sections

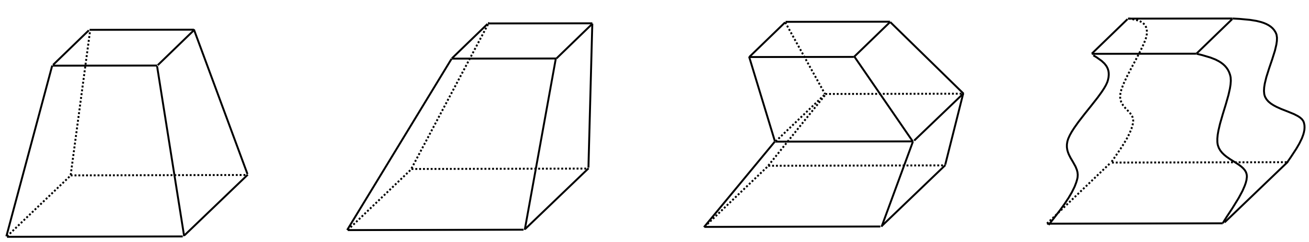
A *cross section* is the 2-dimensionsl shape that results when a solid (3-dimensional) object is intersected by a plane.

Ex: What cross-sectional shapes are possible when a plane intersects a cube?

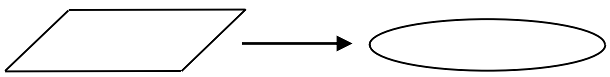


Cavalieri's Principle

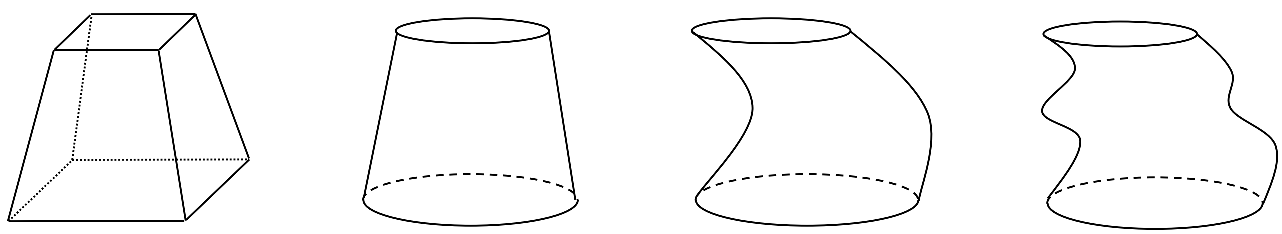
Ex: Suppose we had several thousand rectangular sheets of paper of different sizes stacked in the shape of a truncated pyramid (left). Suppose we then rearrange the *exact same pieces of paper* (stacked in the same order) so that the pyramid "leans" (2^{nd} figure). Then we rearrange them twice more to get the third and fourth figures below. What can we say about the volumes of all four figures?



Ex: Suppose for each rectangular sheet of paper in the previous example, we have a matching sheet in the shape of an oval but having the *same area*.



Let's arrange these sheets into a truncated cone (2^{nd} figure). How will the volume of the cone compare to the volume of the original pyramid?



Now let the papers be rearranged twice more to give the third and fourth figures above. What can we say about the volumes of all four figures?

Cavalieri's Principle: If two figures have the same height and if cross sections parallel to their bases and at the same distance (height) from their bases always have the same area, then the figure have the same volume.

Ex: The oblique triangular pyramid and the right circular cone shown have the same height. Cross sections of each taken at the same height and parallel to the bases have different shapes (obviously) but the *same areas*. Which has the greater volume?