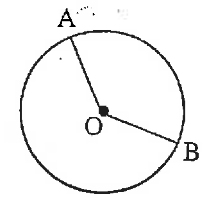
Geometry of a Circle

Measures of ANGLES and ARCS:

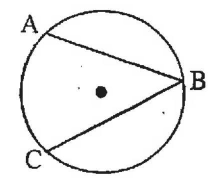
1. Central angles (two radii):

$$m\angle AOB = m\overset{\frown}{AB}$$



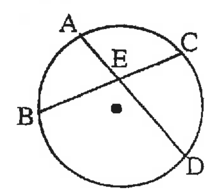
2. Inscribed Angles (two chords with the vertex ON the circle):

$$m\angle ABC = \frac{1}{2} m\overset{\frown}{AC}$$



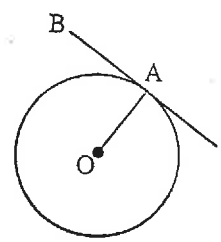
3. Two chords: **INSIDE = HALF THE SUM**

$$m\angle AEB = \frac{1}{2}\left(m\overset{\frown}{AB} + m\overset{\frown}{CD}\right)$$



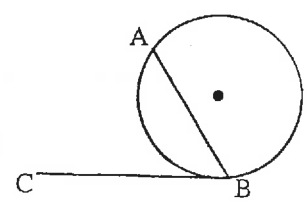
4. Tangent and a radius:

$$m\angle OAB = 90^\circ$$



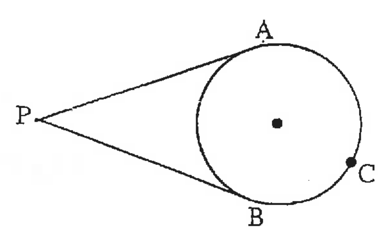
5. Tangent and a chord:

$$m\angle ABC = \frac{1}{2} m\overset{\frown}{AB}$$



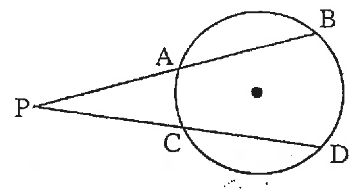
6. Two tangents: **OUTSIDE = HALF THE DIFFERENCE**

$$m\angle P = \frac{1}{2}\left(m\overset{\frown}{ACB} - m\overset{\frown}{AB}\right)$$



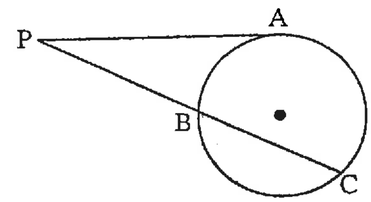
7. Two secants: **OUTSIDE = HALF THE DIFFERENCE**

$$m\angle P = \frac{1}{2}\left(m\widehat{BD} - m\widehat{AC}\right)$$



8. Tangent and secant: **OUTSIDE = HALF THE DIFFERENCE**

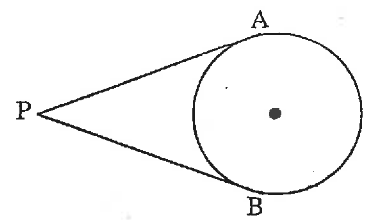
$$m\angle P = \frac{1}{2}\left(m\widehat{AC} - m\widehat{AB}\right)$$



Measures of SECANTS, CHORDS, and TANGENTS:

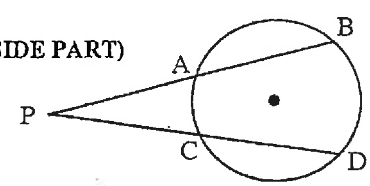
9. Two tangents from the same external point:

$$PA = PB$$



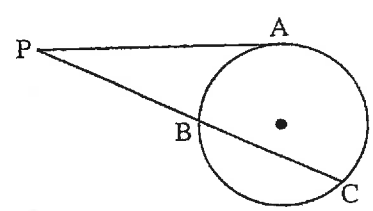
10. Two secants: **(WHOLE)(OUTSIDE PART) = (WHOLE)(OUTSIDE PART)**

$$(PB)(PA) = (PD)(PC)$$



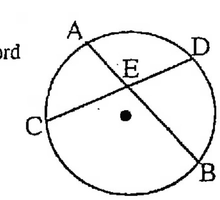
11. Tangent and a secant:

$$(PA)^2 = (PC)(PB)$$



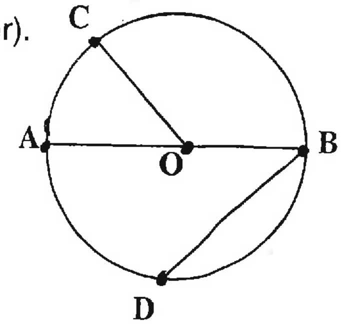
12. Two chords: **(PART)(PART)** of one chord = **(PART)(PART)** of the other chord

$$(AE)(EB) = (DE)(EC)$$



powerpt 4-6-10

Circle Geometry: Angles and Arcs



Circle: The set of points that are equidistant from a fixed point (the center).

Radius: The line segment from the center to a point on the circle.

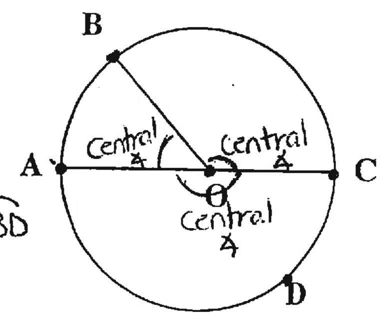
** All radii of the same circle are congruent **
** Circles with congruent radii are congruent.

Radii: $\overline{OA}$, $\overline{OB}$, $\overline{OC}$

Chord: A line segment that connects any point on the circle with any other point on the circle.

CHORDS: $\overline{AB}$, $\overline{BD}$

** A diameter is a chord which cuts a circle into a semicircle. **



Arc: Any portion of the circle's circumference.

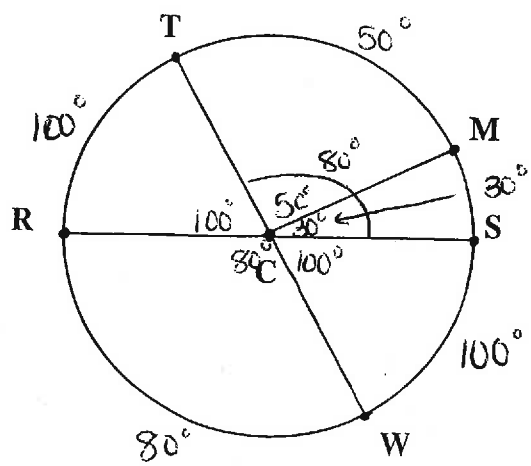
Minor Arc: An arc with a measure less than 180°. $\overset{\frown}{AB}$, $\overset{\frown}{BC}$
Major Arc: An arc with a measure greater than 180°. $\overset{\frown}{ADB}$, $\overset{\frown}{ABD}$

** The sum of the arcs of a circle is 360°.**

Central Angle: An angle whose vertex is the center of the circle and its sides are radii.

$\angle AOB$, $\angle AOC$, $\angle BOC$

Theorem: The measure of a central angle is equal to the measure of the arc that it intercepts.



1. In circle C, $\overline{TW}$ and $\overline{RS}$ are diameters.

   If m$\angle TCS$ = 80° and m$\overset{\frown}{MS}$ = 30°, find

   a. m $\overset{\frown}{TM}$ = 50°

   b. m$\angle MCS$ = 30°

   c. m $\overset{\frown}{TS}$ = 80°

   d. m$\angle SCW$ = 100°

   e. m$\angle RCW$ = 80°

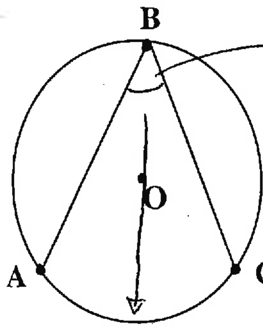
   f. m $\overset{\frown}{MTS}$ = 330°

   g. m $\overset{\frown}{RMW}$ = 280°

4-1-12

Inscribed Angles: An angle whose vertex is on the circle and whose sides are chords of the circle.

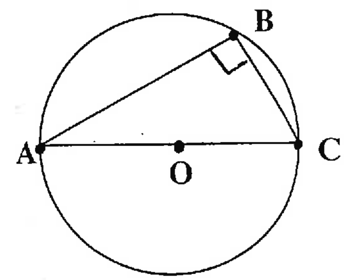
Theorem: The measure of an inscribed angle is equal to $\boxed{\text{one-half}}$ the measure of the arc that intercepts.



inscribed ∡

$m\angle ABC = \frac{1}{2}\overset{\frown}{AC}$

Theorem: An angle inscribed in a semicircle is a right angle.

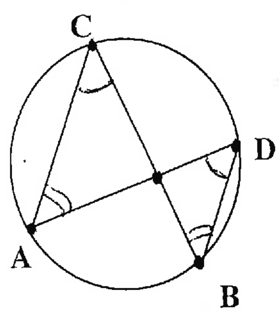


$m\angle ABC = \frac{1}{2}\overset{\frown}{AC}$

$= \frac{1}{2}(180)$

$= 90°$

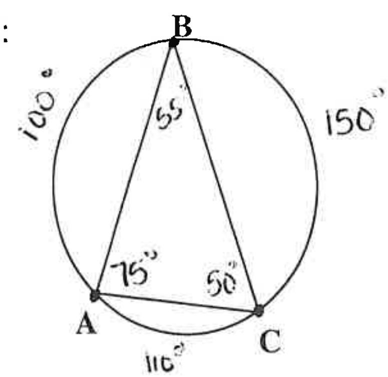
Theorem: Two inscribed angles of a circle that intercept the same arc are congruent.



$m\angle C \cong m\angle D$

$m\angle A \cong m\angle B$

2. ΔABC is inscribed in a circle, $\overset{\frown}{AB} = 100°$ and $m\angle A = 75°$. Find:



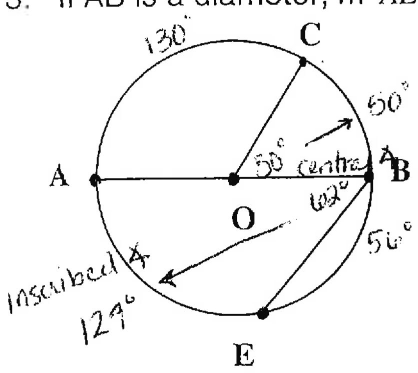
a. $m\angle C$ = 50°

b. $m\angle B$ = 55°

c. $m\overset{\frown}{BC}$ = 150°

d. $m\overset{\frown}{AC}$ = 110°

3. If AB is a diameter, m $\overset{\frown}{AE}$ = 124° and m$\angle COB$ = 50° find:



a. m $\overset{\frown}{BE}$ = 56°

b. m $\overset{\frown}{CB}$ = 50°

c. m$\angle ABE$ = 62°

c. m $\overset{\frown}{AC}$ = 130°

d. m$\angle AOC$ = 130°

Arcs and Chords

Theorems:

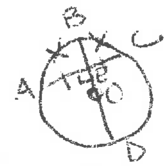
1. In a circle or in congruent circles, congruent central angles have congruent chords.

(Chords) (central ∠'s)

2. In a circle or in congruent circles, congruent arcs have congruent chords.

(chords) (arcs)

3. A diameter perpendicular to a chord bisects the chord and its arc.

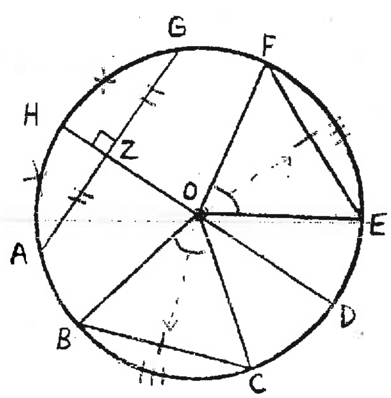


$\overset{\frown}{AB} \cong \overset{\frown}{BC}$

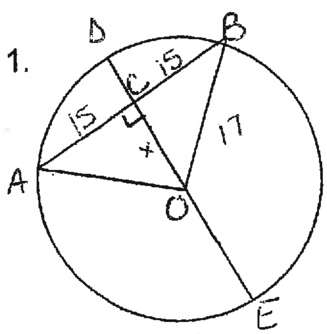
$\overline{AE} \cong \overline{CE}$

** The perpendicular bisector of a chord of a circle passes through the center of the circle.

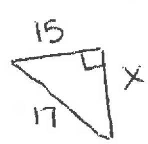
** If two chords of a circle are congruent, they are equidistant from the center of the circle.



1.



$\overline{DE}$ is a diameter of circle O and $\overline{DE} \perp$ chord $\overline{AB}$ at point C. If AB = 30 and OB = 17, find $\overline{OC}$.



$a^2 + b^2 = c^2$

$15^2 + x^2 = 17^2$

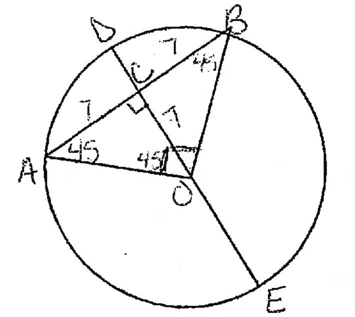
$225 + x^2 = 289$

$x^2 = 64$

$x = 8$

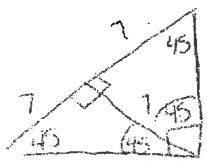
$\boxed{OC = 8}$

2. $\overline{DE}$ is a diameter of circle O and $\overline{DE} \perp$ chord $\overline{AB}$ at point C. If m$\angle AOB$ = 90° and OC = 7, find $\overline{AB}$.

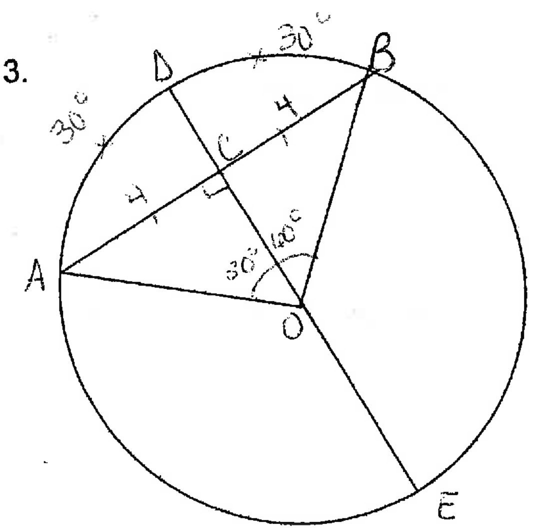


AB = 7+7

$\boxed{AB = 14}$



3.



$\overline{DE}$ is a diameter of circle O and $\overline{DE} \perp$ chord $\overline{AB}$ at point C. If m$\angle AOB$ = 60° and AB = 8, find:

a. m $\widehat{AB}$ 60°

b. m $\widehat{DB}$ 30°

c. m $\angle CAO$ 60°

d. m $\angle AOC$ 30°

e. m$\angle BOC$ 30°

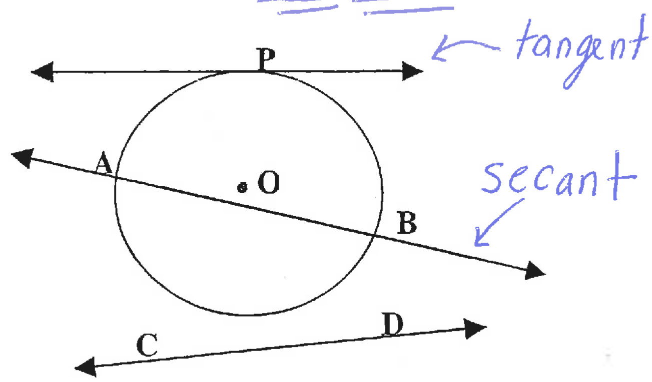
f. m $\widehat{AE}$ 150°

g. AC 4

Tangents and Secants

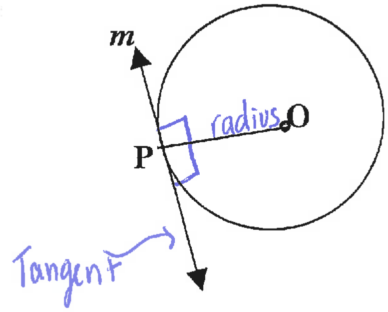
Tangent: a line in the same plane as a circle that intersects the circle at exactly one point.

Secant: a line that intersects a circle at two points.

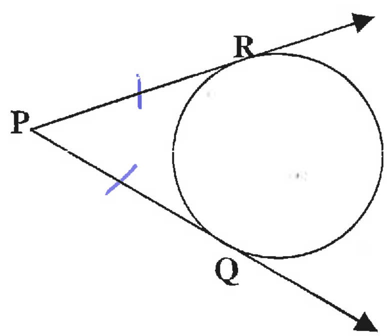


Theorem: If a line is perpendicular to a radius at its point of intersection with a circle, the line is tangent to the circle.

Theorem: If a line is tangent to a circle, the line is perpendicular to the radius drawn to the point of tangency.

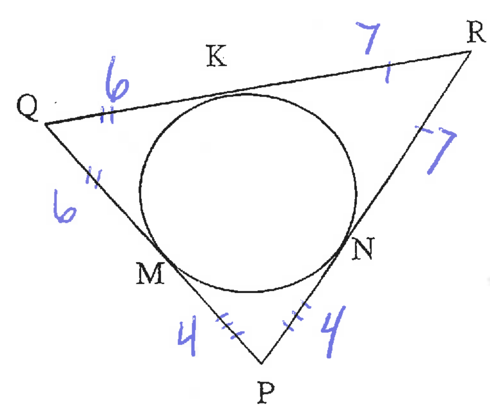


Theorem: Tangent segments drawn to a circle from an external point are congruent.



Examples:

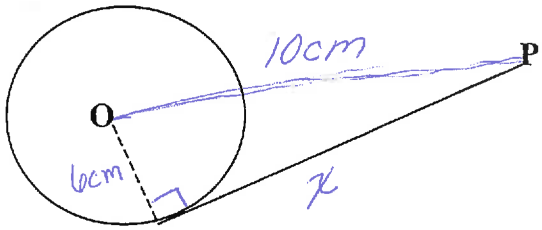
1. If $PM = 4$, $QK = 6$, and $RN = 7$, find the perimeter of ΔPQR.



$P = 6 + 6 + 7 + 7 + 4 + 4$

$= 34$

2. Point P is 10 cm from the center of a circle whose radius is 6 cm. Find the length of the tangent from P to the circle.



$a^2 + b^2 = c^2$

$6^2 + x^2 = 10^2$

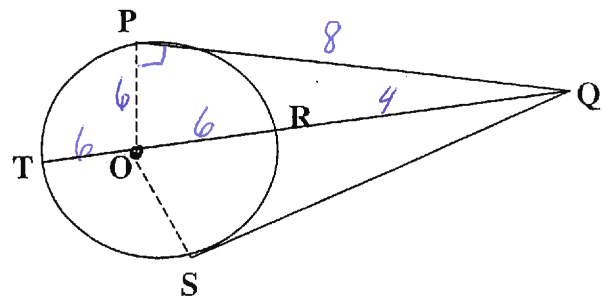
$36 + x^2 = 100$

$-36 \qquad -36$

$\sqrt{x^2} = \sqrt{64}$

$\boxed{x = 8}$

3. $\overline{PQ}$ is tangent to circle O at point P, $\overline{SQ}$ is tangent to circle O at point S, and $\overline{OQ}$ intersects circle O at points T and R.



a) If $RT = 12$ and $RQ = 4$, find:

1. PO 6

2. OQ

$6 + 4 = 10$

3. PQ

$a^2 + b^2 = c^2$

$6^2 + x^2 = 10^2$

$36 + x^2 = 100$

$x^2 = 64$

$x = 8$

4. QS $\cong PQ$

8

b) If $PQ = 2x - 4$, $SQ = x + 4$, and $OP = .5x + 1$

$PQ \cong SQ$

$2x - 4 = x + 4$

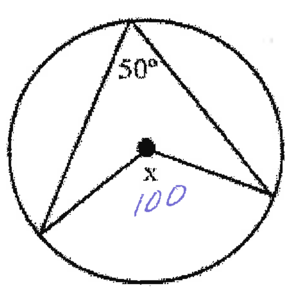
$-x \qquad -x$

$x - 4 = 4$

$+4 \qquad +4$

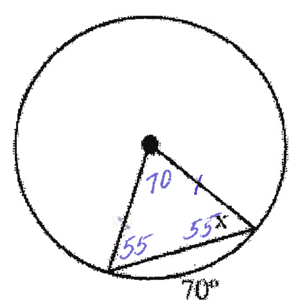
$x = 8$

4.



100

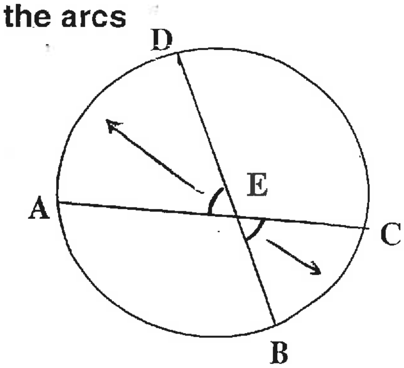
5.



180
-70
$110 \div 2$

Angles Formed by Chords, Tangents, and Secants

Theorem: The measure of an angle formed by two chords intersecting within a circle is equal to one-half the sum of the measures of the arcs intercepted by the angle and by its vertical angle.



1. In Circle O, if $\overset{\frown}{AD} = 45°$ and $\overset{\frown}{BC} = 95°$, find $m\angle DEA$.

not a central ∡!

$m\angle DEA = \frac{1}{2}(\overset{\frown}{AD} + \overset{\frown}{BC})$

$m\angle DEA = \frac{1}{2}(45 + 95)$

$\boxed{m\angle DEA = 70°}$

$m\angle AED = \frac{1}{2}(\overset{\frown}{AD} + \overset{\frown}{BC})$

2. In Circle O, if $\overset{\frown}{AD} = 30°$ and $\angle AED = 55°$, find $\overset{\frown}{BC}$.

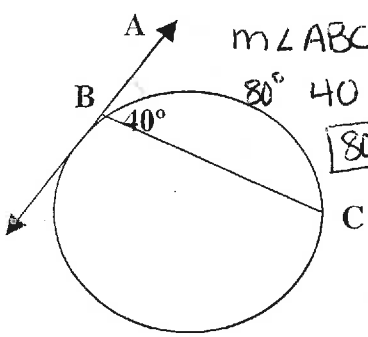
$m\angle DEA = \frac{1}{2}(\overset{\frown}{AD} + \overset{\frown}{BC})$

$2 \cdot \quad 55 = \frac{1}{\cancel{2}}(30 + x) \cdot \cancel{2}$

$110 = 30 + x$

$\boxed{80° = x}$

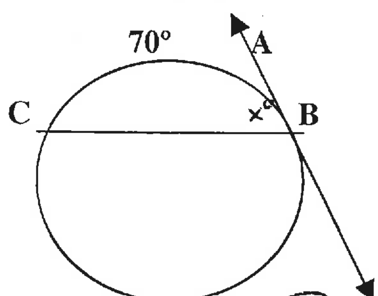
Theorem: The measure of an angle formed by a tangent and a chord intersecting at the point of tangency is equal to one-half the measure of the intercepted arc.



$m\angle ABC = \frac{1}{2}\overset{\frown}{BC}$

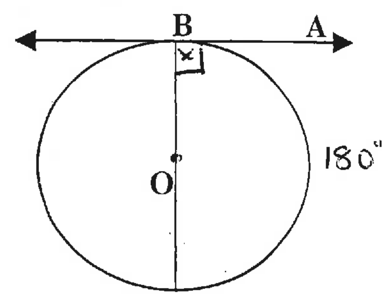
$80° \quad 40 = \frac{1}{2}x$

$\boxed{80° = x}$



$m\angle ABC = \frac{1}{2}\overset{\frown}{BC}$

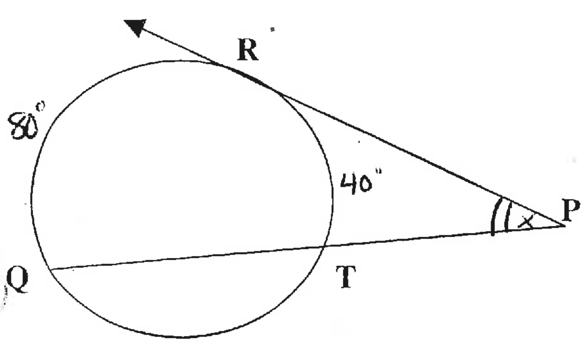
$x = \frac{1}{2}(70) = 35°$



$m\angle ABC = \frac{1}{2}\overset{\frown}{BC}$

$x = \frac{1}{2}(180) = 90°$

Theorem: The measure of an angle formed by a tangent and a secant, or two secants, or two tangents intersecting outside the circle is equal to one-half the difference of the measures of the intercepted arcs.



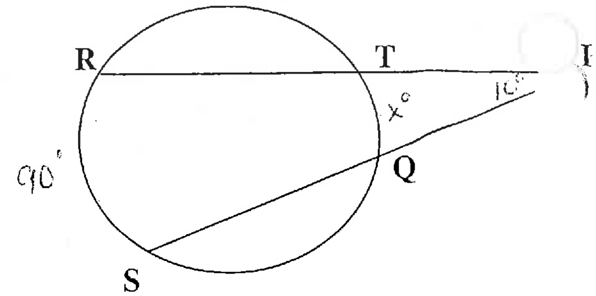
3. Find $\angle P$ if $\overset{\frown}{QR} = 80°$ and $\overset{\frown}{RT} = 40°$

$m\angle P = \frac{1}{2}(\overset{\frown}{RQ} - \overset{\frown}{RT})$

$m\angle P = \frac{1}{2}(80 - 40)$

$\boxed{m\angle P = 20°}$

4. Find $\overset{\frown}{TQ}$ if $\overset{\frown}{RS} = 90°$ and $\angle P = 10°$



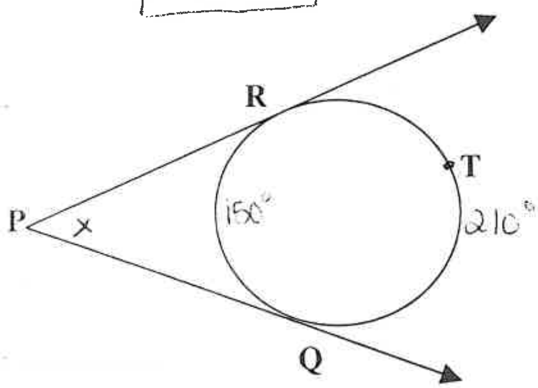
$m\angle P = \frac{1}{2}(\overset{\frown}{RS} - \overset{\frown}{TQ})$

$2 \cdot 10 = \cancel{2} \cdot \cancel{\tfrac{1}{2}}(90 - x)$

$20 = 90 - x$

$-70 = -x$

$\boxed{70° = x}$



5. If $\overset{\frown}{RTQ} = 210°$, find $m\angle P$.

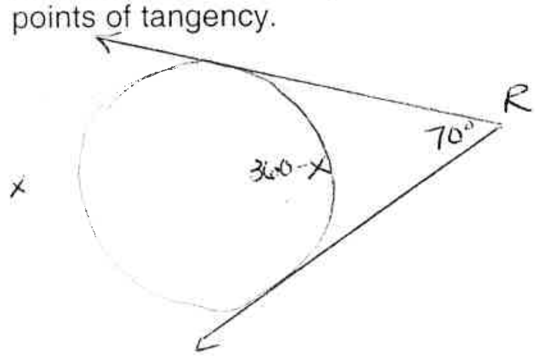
$m\angle P = \frac{1}{2}(\overset{\frown}{RTQ} - \overset{\frown}{RQ})$

$x = \frac{1}{2}(210 - 150)$

$x = \frac{1}{2}(60)$

$\boxed{x = 30°}$

6. Two tangents are drawn to a circle from an external point R such that $m\angle R = 70°$. Find the measure of the major arc and the minor arc into which the circle is divided by the points of tangency.



$\angle R = \frac{1}{2}(x - (360 - x))$

$70 = \frac{1}{2}(x - 360 \boxed{+} x)$

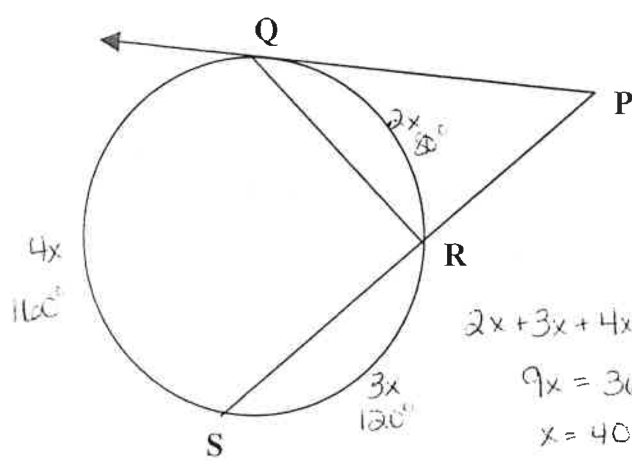
$140 = 2x - 360$

$\frac{500}{2} = \frac{2x}{2}$

$250 = x$

$\boxed{\begin{array}{l}\text{Major Arc} = 250° \\ \text{Minor Arc} = 110°\end{array}}$

7. If $m\overset{\frown}{QR} : m\overset{\frown}{RS} : m\overset{\frown}{SQ} = 2:3:4$, find:



$2x + 3x + 4x = 360$

$9x = 360$

$x = 40$

a) $m\overset{\frown}{QR} = 80°$

b) $m\overset{\frown}{RS} = 120°$

c) $m\angle P = \frac{1}{2}(\overset{\frown}{QS} - \overset{\frown}{QR})$

$= 40°$

d) $m\angle PQR = \frac{1}{2}(\overset{\frown}{QR})$

$= 40°$

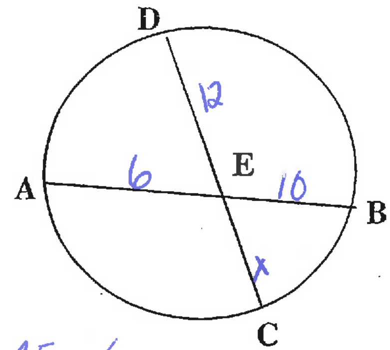
e) $m\angle PRQ = 180° - \angle QRS$

$\boxed{\angle QRS = \frac{1}{2}(QS) = 80°}$ $\rightarrow 180 - 80°$

$= 100°$

Measure of Chords, Tangent Segments, and Secant Segments

Theorem: If two chords intersect within a circle, the product of the measures of the segments of one chord equals the product of the measures of the segments of the other. (Part)(Part) = (Part)(Part)



1. If AE = 6, EB = 10, and ED = 12, find EC. = 5

$(6)(10) = (12)(x)$

$\frac{60}{12} = \frac{12x}{12}$ $x = 5$

2. IF CE = 4, ED = 12, and EB is 2 more than AE, find AE and EB.

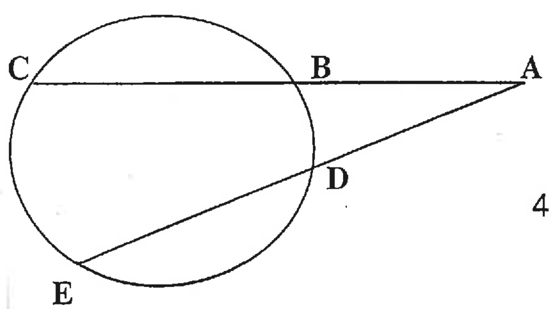
$(4)(12) = (x+2)(x)$ $x^2 + 2x - 48 = 0$

$48 = x^2 + 2x$ $(x+8)(x-6) = 0$ AE = 6

$x = -8$ $x = 6$ EB = 8

Theorem: If two secants intersect outside the circle, the product of the measures of one secant and its external segment equals the product of the measures of the other secant and its external segment.

(whole)(outside) = (whole)(outside)



3. If CA = 12, BA = 4, and DA = 6, find EA, ED, and CB.

$(12)(4) = (x)(6)$ EA = 8

$48 = 6x$ ED = 8 - 6

$x = 8$ = 2

CB = 8

4. If AB = 3, AC = 8, and DE is 2 less than AD, find AE.

$(8)(3) = (2x-2)(x)$ $2(x^2 - x - 12) = 0$

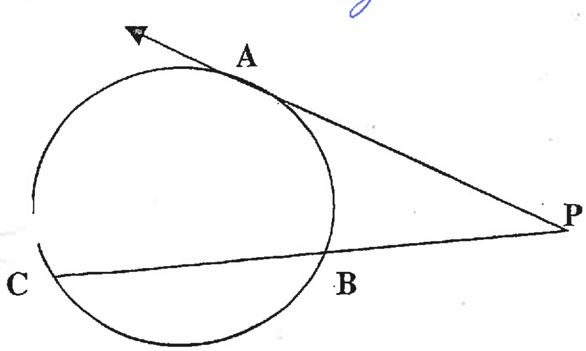
$24 = 2x^2 - 2x$ $(x+3)(x-4)$

$2x^2 - 2x - 24 = 0$ $\cancel{x = -3}$ $x = 4$

AE = x + x - 2
= 6

Theorem: If a tangent and a secant are drawn to a circle from an external point, then the square of the measure of the tangent is equal to the product of the measures of the secant and its external segment.

$(\text{Tangent})^2 = (\text{whole})(\text{outside})$



3. If PB = 3, BCC =9, find PA.

$x^2 = 12(3)$

$x^2 = 36$ $\boxed{x = 6}$

4. If PB:BC = 1:3, and PA = 4, find PB. = 4

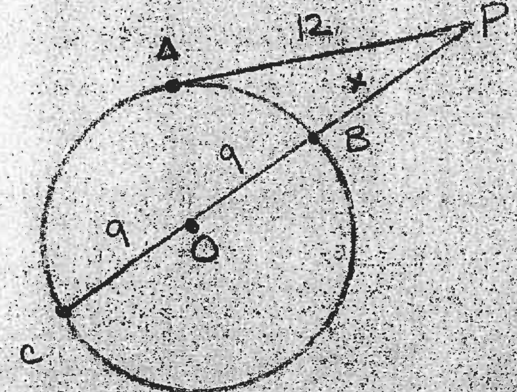
$4^2 = (4x)(1x)$

$16 = 4x^2$

$4 = x^2$ $x = 4$

Word Problems Involving Segments

1. From point P, $\overline{PA}$ is drawn tangent to circle O at A. A secant drawn from P through point O intersects the circle at points B and C. If $PA = 12$ and the length of a radius of circle O is 9, find the length of the external segment of secant $\overline{PBC}$.



$(PA)^2 = (PBC)(PB)$

$12^2 = (x+18)(x)$

$144 = x^2 + 18x$

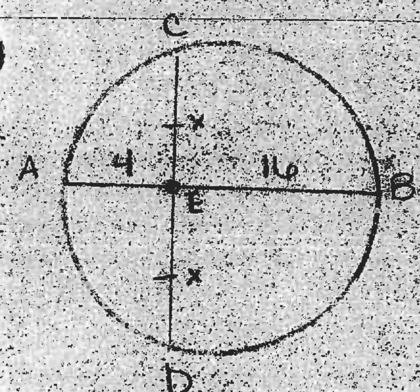
$0 = x^2 + 18x - 144$

$0 = (x+24)(x-6)$

~~x = -24~~ reject | $x = 6$

External Segment of $\overline{PBC} = 6$

2. Chord $\overline{AB}$ bisects chord $\overline{CD}$ in the interior of circle O at point E. If $AE = 4$ and $AB = 20$, find the length of $\overline{CD}$.



$(AE)(EB) = (CE)(ED)$

$4 \cdot 16 = x \cdot x$

$64 = x^2$

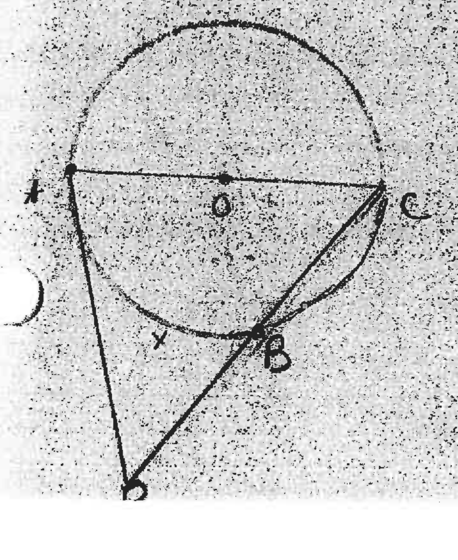
$8 = x$

$CE = 8$

$ED = 8$

$\boxed{CD = 16}$

3. Point P lies outside circle O, which has a diameter of $\overline{AOC}$. The angle formed by tangent $\overline{PA}$ and secant $\overline{PBC}$ measures 30°. Find the number of degrees in the measure of minor arc CB.



$\angle P = \frac{1}{2}(m\overset{\frown}{AC} - m\overset{\frown}{AB})$

$30 = \frac{1}{2}(180 - x)$

$60 = 180 - x$

$-120 = -x$

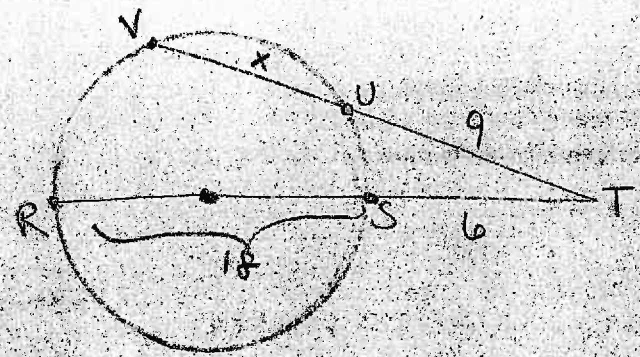
$120° = x$

$\overset{\frown}{AB} = 120°$

$\overset{\frown}{CB} = 180 - 120°$

$\boxed{\overset{\frown}{CB} = 60°}$

4. In a circle, diameter RS is extended through S to an external point T. Secant $\overline{TUV}$ is then drawn. If $RS = 18$, $ST = 6$, and $UT = 9$, find TV.



$(VUT)(UT) = (RST)(ST)$

$(x+9)(9) = (24)(6)$

$9x + 81 = 144$

$9x = 63$

$x = 7$

$VU = 7$

$\boxed{VUT = 16}$

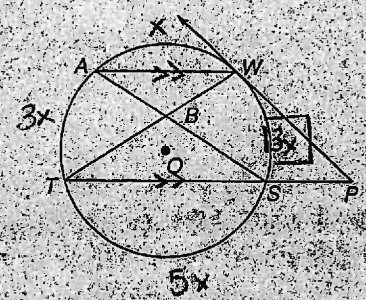
5. In circle O, tangent $\overrightarrow{PW}$ and $\overline{PST}$ are drawn. Chord $\overline{WA}$ is parallel to chord $\overline{ST}$. Chords $\overline{AS}$ and $\overline{WT}$ intersect at point B. If $m\overarc{WA} : m\overarc{AT} : m\overarc{ST} = 1:3:5$, find:

a. $m\angle TBS$ $\boxed{90°}$

b. $m\angle TWP$ $\boxed{120°}$

c. $m\angle WPT$ $\boxed{15°}$

d. $m\angle ASP$ $\boxed{135°}$



* parallel chords cut off $\cong$ arcs !!

X:

$x + 3x + 5x + 3x = 360$

$12x = 360$

$x = 30°$

$\overarc{AW} = 30°$

$\overarc{AT} \cong \overarc{WS} = 90°$

$\overarc{TS} = 150°$

(a.) $\angle TBS = \frac{1}{2}(\overarc{AW} + \overarc{ST})$

$= \frac{1}{2}(30 + 150)$

$= \frac{1}{2}(180)$

$= 90°$

(b.) $\angle TWP = \frac{1}{2}(\overarc{TSW}$

$= \frac{1}{2}(240)$

$= 120°$

(c.) $\angle WPT = \frac{1}{2}(\overarc{WAT} - \overarc{WS})$

$= \frac{1}{2}(120° - 90°)$

$= \frac{1}{2}(30°)$

$= 15°$

(d.) $m\angle ASP = 180 - \angle AST$

$\angle AST = \frac{1}{2}(\overarc{AT})$

$= \frac{1}{2}(90)$

$= 45°$

$m\angle ASP = 180 - 45°$

$= 135°$