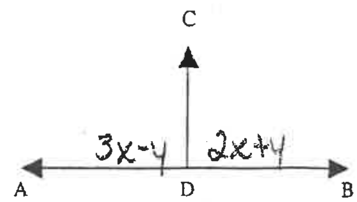
Algebraic Applications of Proving Lines Perpendicular

1. Let D be a point on $\overleftrightarrow{AB}$ between A and B.

If $\overleftrightarrow{CD} \perp \overleftrightarrow{AB}$, m∠ADC = $3x - y$, and m∠CDB = $2x + y$, **find the value of x and y.**



Means both angles are 90

$$\begin{aligned} 3x - y &= 90 \\ + \; 2x + y &= 90 \\ \hline 5x &= 180 \\ \boxed{x = 36} \end{aligned}$$

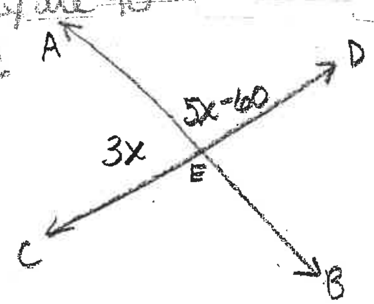
$$\begin{aligned} 2x + y &= 90 \\ 2(36) + y &= 90 \\ 72 + y &= 90 \\ \boxed{y = 18} \end{aligned}$$

2.

$\overleftrightarrow{AB}$ intersects $\overleftrightarrow{CD}$ at E, m∠AEC = $3x$ and m∠AED = $5x - 60$.

Show that $\overleftrightarrow{AB}$ is perpendicular to $\overleftrightarrow{CD}$.

Don't Assume they are 90° Prove this!



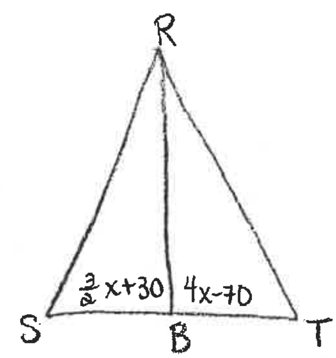
$$\begin{aligned} 3x + 5x - 60 &= 180 \\ 8x - 60 &= 180 \\ 8x &= 240 \\ x &= 30 \end{aligned}$$

$\boxed{\angle AEC = 3(30) = 90}$

$\boxed{\angle AED = 5(30) - 60 = 90}$

Since ∠AEC and ∠AED are both right angles, $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$.

3. In ΔRST, a line drawn from vertex R intersects $\overline{ST}$ in B. If m∠SBR = $\frac{3}{2}x + 30$ and m∠TBR = $4x - 70$, **show that $\overline{RB}$ is an altitude in ΔRST.**



$$\begin{aligned} \tfrac{3}{2}x + 30 + 4x - 70 &= 180 \\ 5.5x - 40 &= 180 \\ 5.5x &= 220 \\ x &= 40 \end{aligned}$$

$$\begin{aligned} \angle SBR &= \tfrac{3}{2}(40) + 30 \\ &= 60 + 30 \\ &= 90 \end{aligned}$$

$$\begin{aligned} \angle TBR &= 4(40) - 70 \\ &= 160 - 70 \\ &= 90 \end{aligned}$$

* ∠SBR and ∠TBR are right angles → Since $\overline{RB} \perp \overline{ST}$, then $\overline{RB}$ is an altitude in ΔRST

Proving Lines Perpendicular

Theorem: If two intersecting lines form congruent adjacent angles then they are perpendicular.

To prove 2 lines or segments are perpendicular:

1. Show when the 2 lines or segments intersect, they form right angles.

OR

2. Show when the 2 lines or segments intersect, they form congruent adjacent angles.

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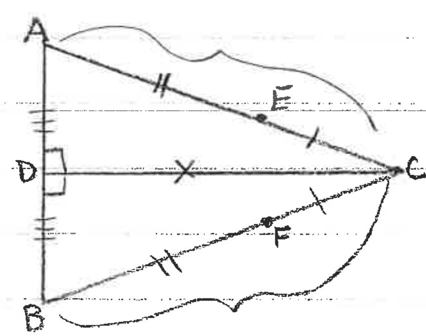
1. Given: $\overline{CE} \cong \overline{CF}$
   $\overline{EA} \cong \overline{FB}$
   D is the midpoint of $\overline{AB}$

   Prove: $\overline{AB} \perp \overline{CD}$

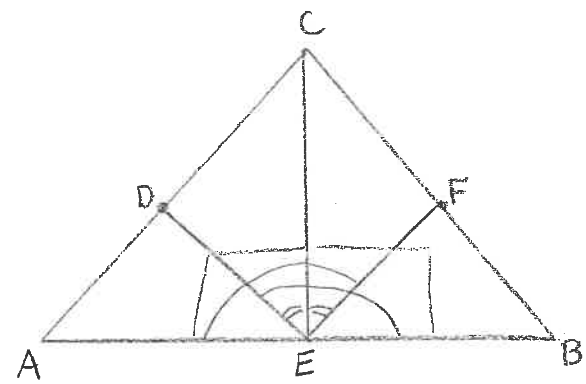


Statements	Reasons
1. $\overline{CE} \cong \overline{CF}$, $\overline{EA} \cong \overline{FB}$	1. Given
S 2. $\overline{AC} \cong \overline{BC}$	2. Addition Postulate
3. D is the midpoint of $\overline{AB}$	3. Given
S 4. $\overline{AD} \cong \overline{BD}$	4. A midpoint divides a segment into 2 $\cong$ segme
S 5. $\overline{CD} \cong \overline{CD}$	5. Reflexive Postulate
6. $\triangle ADC \cong \triangle BDC$	6. S.S.S. $\cong$ S.S.S.
7. $\angle ADC \cong \angle BDC$	7. If 2 $\triangle$'s are $\cong$, then their corresponding parts are $\cong$.
8. $\overline{AB} \perp \overline{CD}$	8. If 2 intersecting lines form $\cong$ adjacer angles, then they are perpendicula

② Given: $\angle AEF \cong \angle BED$

$\angle CEF \cong \angle CED$

Prove: $\overline{AB} \perp \overline{CE}$



Statements	Reasons
① $\angle AEF \cong \angle BED$	① Given
② $\angle CEF \cong \angle CED$	② Given
③ $\angle AEC \cong \angle BEC$	③ Subtraction Postulate
④ $\overline{AB} \perp \overline{CE}$	④ If 2 intersecting lines form $\cong$ adjacent ∠'s, then they are ⊥

Proving Triangles Congruent Using Hypotenuse – Leg (Hyp – Leg)

Two right triangles are congruent if their hypotenuses and a pair of legs are respectively congruent.

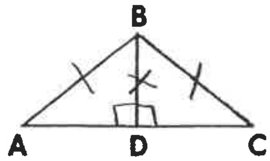
If hy-leg ≅ hy-leg, the right triangles are congruent.

you can only use this postulate if there are RIGHT Δ's !

1. *Given:* $\triangle ABC$, $\overline{BD} \perp \overline{AC}$, $\overline{AB} \cong \overline{CB}$.

   Prove: $\triangle ADB \cong \triangle CDB$.

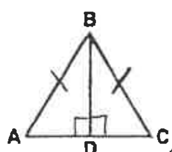
   Plan: Show that $\triangle ADB$ and $\triangle CDB$ are right triangles. Then use hy. leg ≅ hy. leg to prove them congruent.



	Statements	Reasons
	① $\triangle ABC$, $\overline{BD} \perp \overline{AC}$	① Given
	② $\angle ADB$ and $\angle CDB$ are right ∠'s	② ⊥ segments form right ∠'s
	③ $\angle ADB \cong \angle CDB$	③ All right ∠'s are ≅
	④ $\triangle ADB$ and $\triangle CDB$ are right Δ's	④ If a Δ has a right ∠, then it is a right Δ.
leg	⑤ $\overline{BD} \cong \overline{BD}$	⑤ Reflexive Postulate
hyp	⑥ $\overline{AB} \cong \overline{BC}$	⑥ Given
	⑦ $\triangle ADB \cong \triangle CDB$	⑦ hyp.-leg ≅ hyp. leg

2. *Given:* Isosceles $\triangle ABC$, altitude $\overline{BD}$ to base $\overline{AC}$.

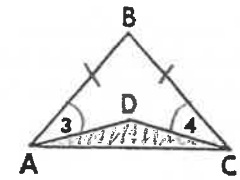
   Prove: $\triangle ABD \cong \triangle CBD$



	Statements	Reasons
	① Isosceles $\triangle ABC$, altitude $\overline{BD}$ to base $\overline{AC}$	① Given
hyp	② $\overline{AB} \cong \overline{BC}$	② If a Δ is isosceles, then it has 2 ≅ sides.
	③ $\overline{BD} \perp \overline{AC}$	③ If a segment is an altitude, then it is ⊥ to the opposite side.
	④ $\angle BDA$ and $\angle BDC$ are right ∠'s	④ ⊥ segments form right ∠'s
	⑤ $\angle BDA \cong \angle BDC$	⑤ All right ∠'s are ≅
	⑥ $\triangle ABD$ and $\triangle CBD$ are right Δ's	⑥ If a Δ has a right ∠, then it is a right Δ.
leg	⑦ $\overline{BD} \cong \overline{BD}$	⑦ Reflexive Postulate
	⑧ $\triangle ABD \cong \triangle CBD$	⑧ hyp.-leg ≅ hyp.-leg

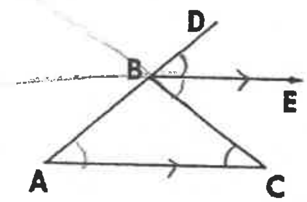
More Isosceles Triangle Proofs

1. If $\overline{AB} \cong \overline{BC}$ and $\angle 3 \cong \angle 4$, prove that $\triangle ADC$ is isosceles.



Statements	Reasons
1) $\overline{AB} \cong \overline{BC}$, $<3 \cong <4$	1) given
2) $<BAC \cong <BCA$	2) If 2 sides of a Δ are $\cong$ angles opp those sides are $\cong$
3) $<BAC - <3 = <BCA - <4$	3) Subtraction
4) $<DAC = <BAC - <3$ $<DCA = <BCA - <4$	4) Partition
5) $<DAC = <DCA$	5) substitution
6) Δ ADC is isosceles	6) An Isoscles Δ has 2 $\cong$ base angles

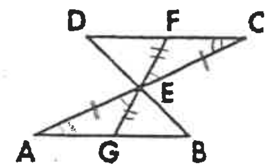
2. *Given:* $\triangle ABC$, $\overline{ABD}$, $\overrightarrow{BE}$ bisects $\angle DBC$, $\overrightarrow{BE} \parallel \overline{AC}$.
Prove: $\overline{AB} \cong \overline{CB}$.



Statements	Reasons
1) $\overrightarrow{ABD}$, $\overrightarrow{BE}$ bisects $<DBC$, $\overline{BE} \parallel \overline{AC}$	1) given
2) $<DBE \cong <CBE$	2) A bisector divides an angle into 2 $\cong$ angles
3) $<BCA \cong <CBE$	3) If 2 lines are parallel, the alternate interior angles are $\cong$
4) $<BAC \cong <DBE$	4) If 2 lines are parallel, the corresponding angles are $\cong$
5) $<BAC \cong <BCA$	5) Transitive
6) $\overline{AB} \cong \overline{CB}$	6) If 2 angles are $\cong$ the sides opp those angles are $\cong$

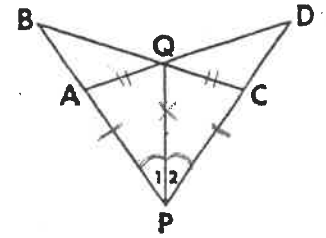
Using Two Pairs of Congruent Triangles

1. *Given:* $\overline{AEC}$, $\overline{BED}$, and $\overline{GEF}$; $\overline{AE} \cong \overline{CE}$, $\overline{FE} \cong \overline{GE}$.
Prove: a. $\triangle FEC \cong \triangle GEA$. b. $\angle C \cong \angle A$. c. $\triangle DEC \cong \triangle BEA$.



	Statements	Reasons
	1) $\overline{AEC}$, $\overline{BED}$, $\overline{GEF}$	1) given
	2) $\overline{AE} \cong \overline{CE}$, $\overline{FE} \cong \overline{GE}$	2) given
	3) $\angle AEG \cong \angle CEF$	3) All vertical angles are $\cong$
a)	4) $\Delta FEC \cong \Delta GEA$	4) SAS $\cong$ SAS
b)	5) $\angle C \cong \angle A$	5) CPCTC
	6) $\angle DEF \cong \angle BEG$	6) All vertical angles are $\cong$
	6.5) $\angle DEF + \angle CEF = \angle BEG + \angle AEG$	6.5) Addition
	7) $\angle DEC = \angle DEF + \angle CEF$ $\angle AEB = \angle BEG + \angle AEG$	7) Partition
	8) $\angle DEC \cong \angle AEB$	8) substitution
	9) $\Delta DEC \cong \Delta BEA$	9) ASA $\cong$ ASA

2. *Given:* $\overline{PQ}$, $\overline{PAB}$, $\overline{PCD}$, $\overline{AQD}$, and $\overline{CQB}$. $\angle 1 \cong \angle 2$ and $\overline{AP} \cong \overline{CP}$.
Prove: $\overline{QB} \cong \overline{QD}$.



Statements	Reasons
1) $\overline{PQ}$, $\overline{PAB}$, $\overline{PCD}$, $\overline{AQD}$ and $\overline{CQB}$	1) given
2) $\angle 1 \cong \angle 2$	2) given
3) $\overline{AP} \cong \overline{CP}$	3) given
4) $\overline{PQ} \cong \overline{PQ}$	4) Reflexive
5) $\triangle APQ \cong \triangle CPQ$	5) SAS $\cong$ SAS
6) $\overline{AQ} \cong \overline{CQ}$	6) CPCTC
7) $\angle PAQ \cong \angle PCQ$	7) CPCTC
8) $\angle BAQ$ and $\angle PAQ$ are supp $\angle DCQ$ and $\angle PCQ$ are supp	8) If 2 angles form a linear pair they are supplementary
9) $\angle BAQ \cong \angle DCQ$	9) Supplements of $\cong$ angles are $\cong$
10) $\angle BQA \cong \angle DQC$	10) All vertical angles are $\cong$
11) $\triangle ABQ \cong \triangle CDQ$	11) ASA $\cong$ ASA
12) $\overline{QB} \cong \overline{QD}$	12) CPCTC

Parallel Lines in Proofs

To prove two lines that are cut by a transversal are parallel, prove that any ONE of the following is true:

1. A pair of alternate interior angles are congruent.
2. A pair of corresponding angles are congruent.
3. A pair of interior angles on the same side of the transversal are supplementary.

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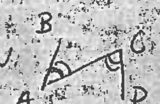
1. Given: Quadrilateral ABCD, $\overline{BC} \cong \overline{DA}$

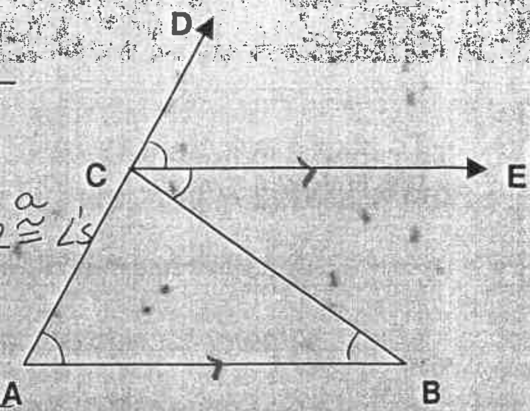
   $\overline{BC} \parallel \overline{DA}$

   Prove: $\overline{AB} \parallel \overline{CD}$ (show )

Statements	Reasons
1. Quadrilateral ABCD, $\overline{BC} \cong \overline{DA}$ $\overline{BC} \parallel \overline{DA}$	1. Given
2. $\angle BCA \cong \angle DAC$	2. If 2 ∥ lines are cut by a transversal, alternate interior ∡'s are ≅
3. $\overline{AC} \cong \overline{AC}$	3. Reflexive Postulate
4. $\triangle BCA \cong \triangle DAC$	4. SAS ≅ SAS
5. $\angle BAC \cong \angle DCA$	5. Corresponding parts of ≅ △'s are ≅
6. $\overline{AB} \parallel \overline{CD}$	6. If 2 lines are cut by a trans and form ≅ alternate interior ∠'s, they are ∥ lines.

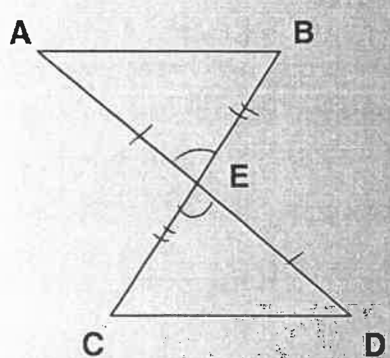
2.

Statements	Reasons
1. $\overrightarrow{CE}$ bisects $\angle DCB$	1. Given
2. $\angle DCE \cong \angle ECB$	2. A bisector cuts a segment into 2 $\cong$ $\angle$'s
3. $\overline{CE} \parallel \overline{AB}$	3. Given
4. $\angle ECB \cong \angle B$	4. If 2 $\parallel$ lines are cut by a transversal, alt. int $\angle$'s are $\cong$
5. $\angle DCE \cong \angle B$	5. Substitution Postulate
6. $\angle DCE \cong \angle A$	6. If 2 $\parallel$ lines are cut by a transversal, corresponding $\angle$'s are $\cong$.
7. $\angle A \cong \angle B$	7. Substitution Postulate.



If $\overline{CE}$ bisects $\angle DCB$ and $\overline{CE} \parallel \overline{AB}$, prove $\angle A \cong \angle B$.

3.



Given: $\overline{AD}$ and $\overline{BC}$ bisect each other at E

Prove: $\overline{AB} \parallel \overline{CD}$,

Statements	Reasons
1. $\overline{AD}$ & $\overline{BC}$ bisect each other at E.	1. Given
2. $\overline{AE} \cong \overline{DE}$ $\overline{BE} \cong \overline{CE}$	2. A bisector cuts a segment into 2 $\cong$ segments
3. $\angle AEB \cong \angle DEC$	3. Vertical $\angle$'s are $\cong$
4. $\triangle AEB \cong \triangle DEC$	4. SAS $\cong$ SAS
5. $\angle A \cong \angle D$ (OR $\angle C \cong \angle B$)	5. Corresponding parts of $\cong$ $\triangle$'s are $\cong$
6. $\overline{AB} \parallel \overline{CD}$	6. If 2 lines are cut by a transversal & form $\cong$ alt int $\angle$'s they are