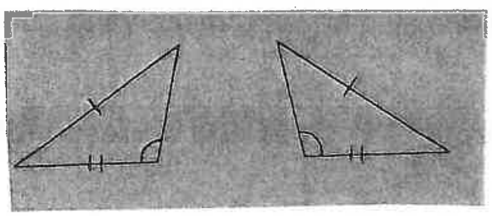
SAS Postulate

Two triangles are congruent if **two sides and the included angle** of one triangle are congruent respectively to two sides and the included angle of the other triangle.

Note: The angle must be included between the two congruent sides!!

Examples: Is the given information sufficient to prove the triangles are congruent?

1.



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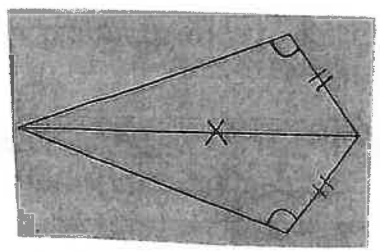

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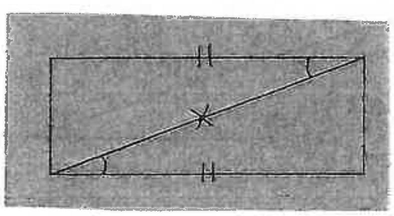

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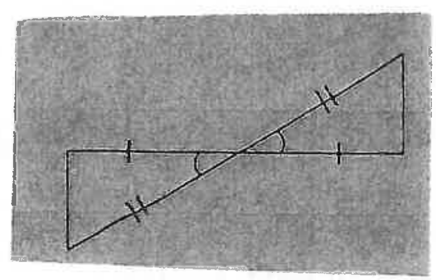
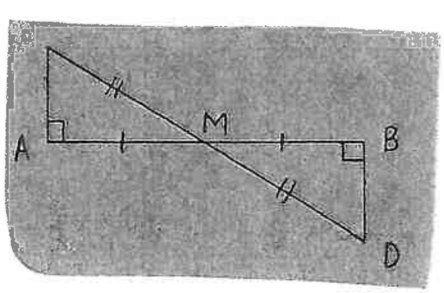


Name the pair of corresponding sides or pair of corresponding angles that would be needed in order to prove the triangles are congruent by SAS.

5.



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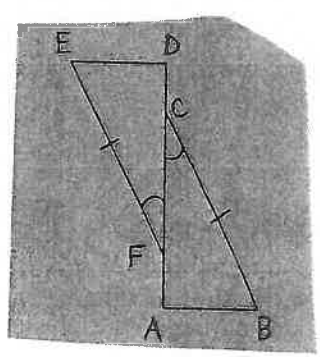

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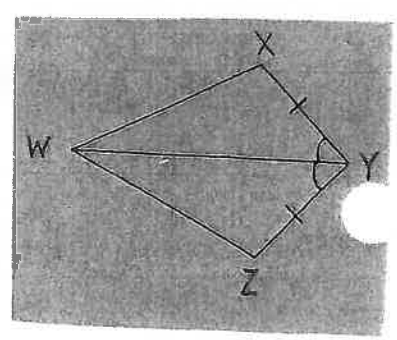


Congruent Triangles and Polygons

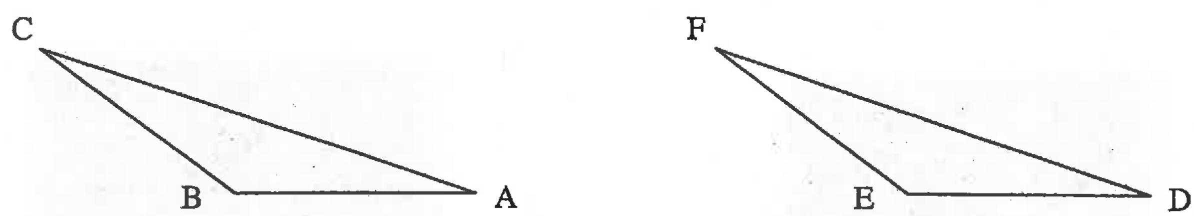
Congruent Polygons: When two polygons can be moved in such a way that the sides and angles of one polygon fit exactly upon the sides and angles of a second polygon (polygons that have the same size and same shape).

The order of the vertices is a one-to-one correspondence.

If $ABCD \cong EFGH$, then A corresponds to E
B corresponds to F
C corresponds to G
D corresponds to H

In addition to corresponding vertices, congruent polgons also have corresponding sides and angles.

Note: If two polygons are congruent, then corresponding sides and corresponding angles are congruent.

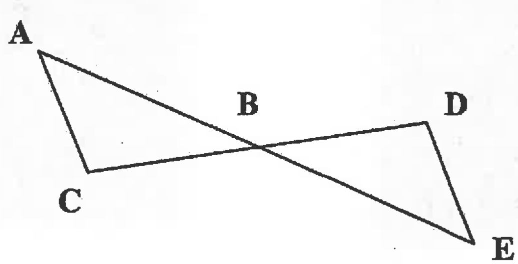


If $\Delta ABC \cong \Delta DEF$, ________________ ________________

________________ ________________

________________ ________________

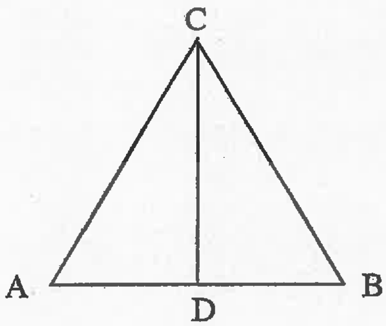
Ex) Make 6 conclusions if $\Delta ABC \cong \Delta EBD$.



Proofs and the SAS Postulate

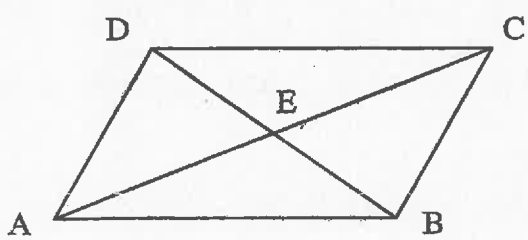
1. Given: $\overline{AC} \cong \overline{BC}$
   $\overline{CD}$ bisects $\angle ACB$

   Prove: $\Delta ACD \cong \Delta BCD$



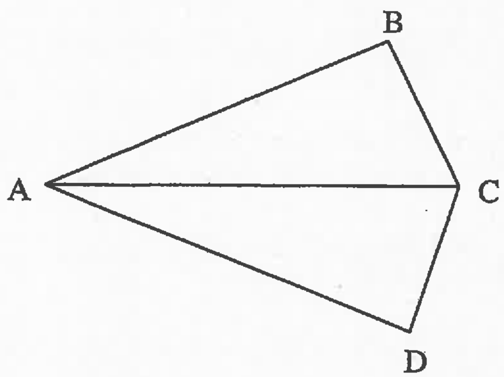
2. Given: $\overline{DB}$ and $\overline{AC}$ bisect each other at E

   Prove: $\Delta AEB \cong \Delta CED$



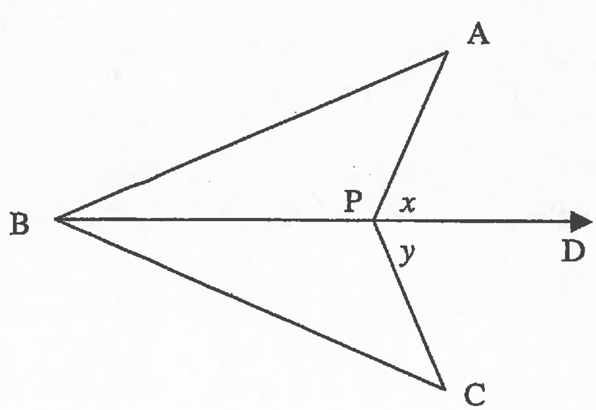
3. Given: $\overline{AB} \perp \overline{BC}$
$\overline{AD} \perp \overline{DC}$
$\overline{AB} \cong \overline{AD}$
$\overline{BC} \cong \overline{DC}$

Prove: $\Delta ABC \cong \Delta ADC$



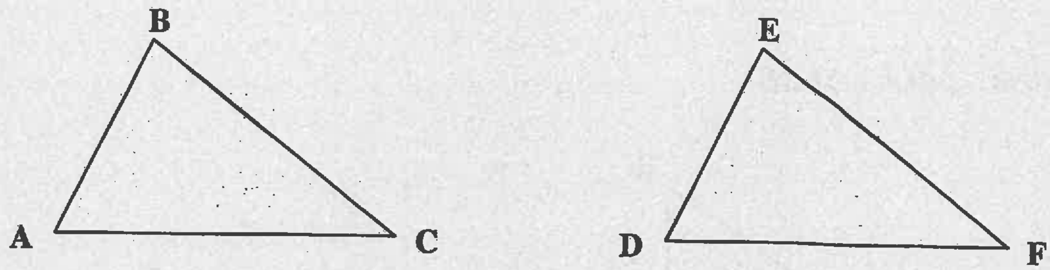
4. Given: $\overline{AP} \cong \overline{CP}$
$\angle x \cong \angle y$
$\overline{BD}$ is a straight line

Prove: $\Delta BAP \cong \Delta BCP$



ASA Postulate

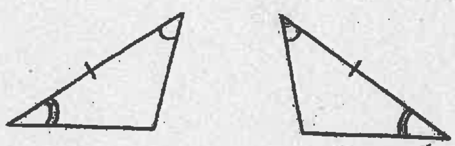
Two triangles are congruent if **two angles and the included side** of one triangle are congruent respectively to two angles and the included side of the other triangle.



Note: The side must be included between the two congruent angles!!

Examples: Is the given information sufficient to prove the triangles are congruent by ASA?

1.



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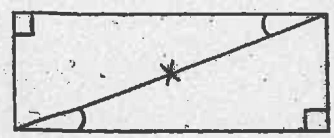
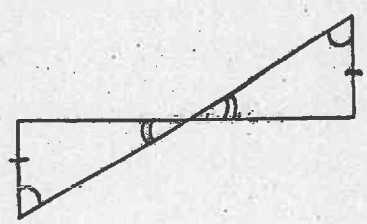


3.



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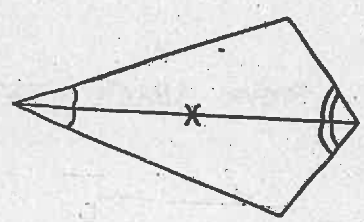
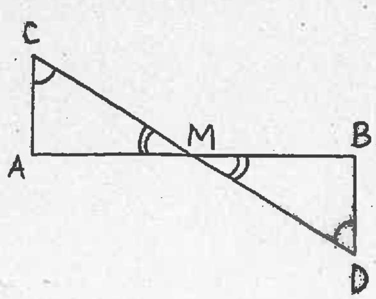


Name the pair of corresponding sides or pair of corresponding angles that would be needed in order to prove the triangles are congruent by ASA.

5.



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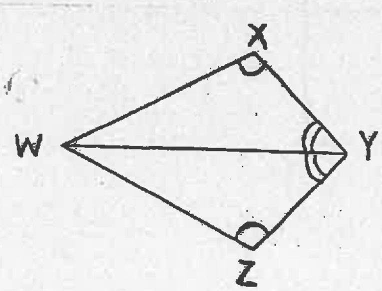

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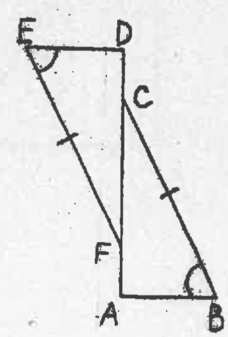
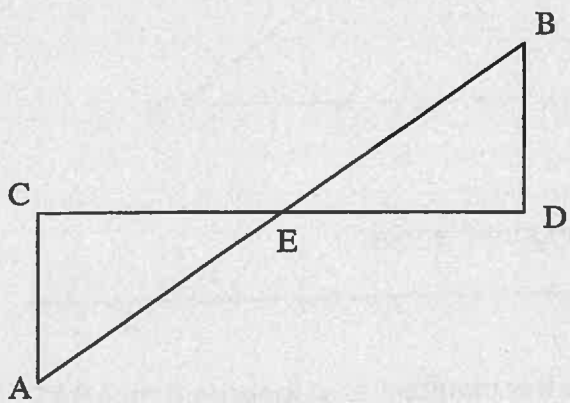


Proofs:

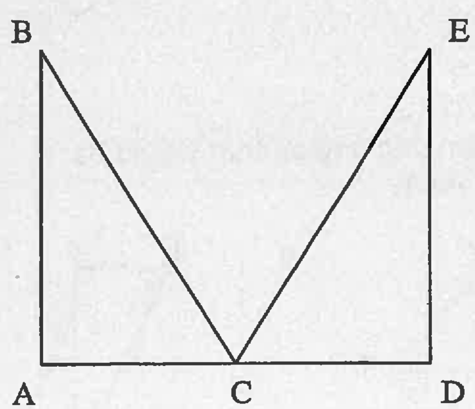
1. Given: $\overline{BA}$ bisects $\overline{CD}$
   $\overline{AC} \perp \overline{CD}$
   $\overline{BD} \perp \overline{CD}$

   Prove: $\Delta ACE \cong \Delta BDE$



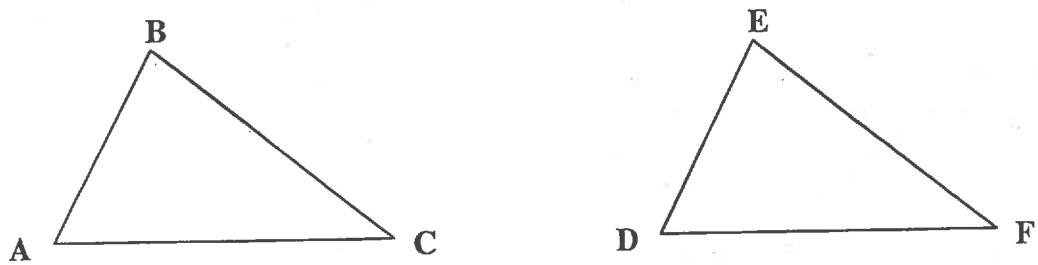
2. Given: C is the midpoint of $\overline{AD}$
   $\overline{BA} \perp \overline{AD}$, $\overline{ED} \perp \overline{DA}$
   $\angle BCA \cong \angle ECD$

   Prove: $\Delta BAC \cong \Delta EDC$



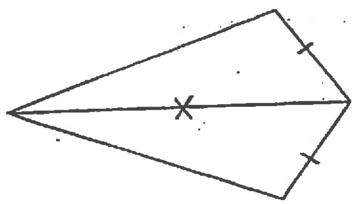
SSS Postulate

Two triangles are congruent if **the three sides** of one triangle are congruent respectively to the three sides of the other triangle.

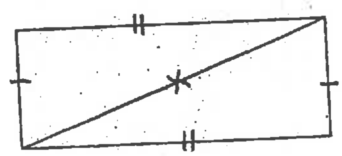


Examples: Is the given information sufficient to prove the triangles are congruent by SSS?

1.

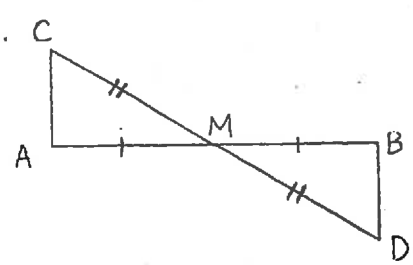


2.



Name the pair of corresponding sides or pair of corresponding angles that would be needed in order to prove the triangles are congruent by ASA.

3.



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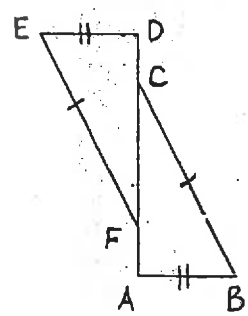
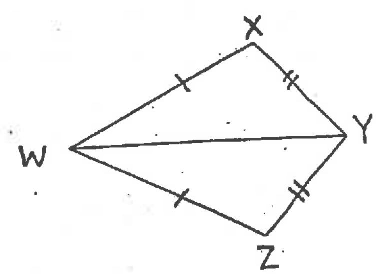


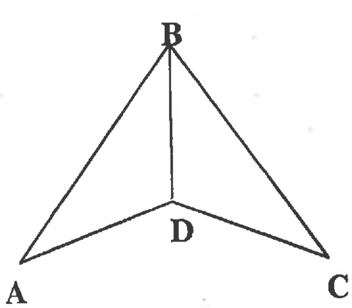
5.



Proofs:

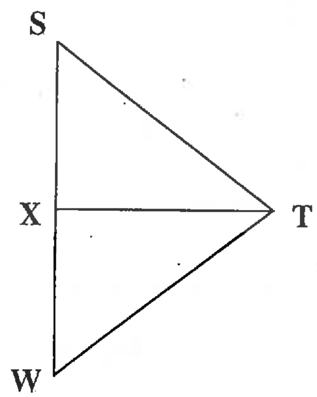
1. Given: $\overline{AB} \cong \overline{CB}$
   $\overline{AD} \cong \overline{CD}$

   Prove: $\Delta ABD \cong \Delta CBD$



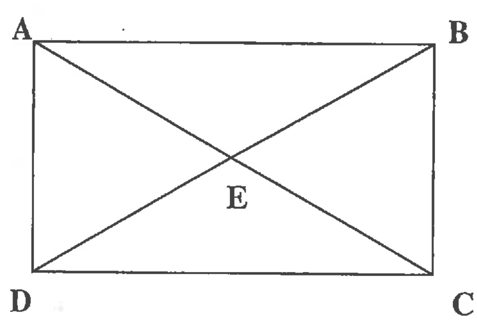
2. Given: x is the midpoint of $\overline{SW}$
$\overline{ST} \cong \overline{WT}$

Prove: $\Delta STX \cong \Delta WTX$



3. Given: $\overline{AC}$ and $\overline{BD}$ bisect each other at E
$\overline{AD} \cong \overline{BC}$

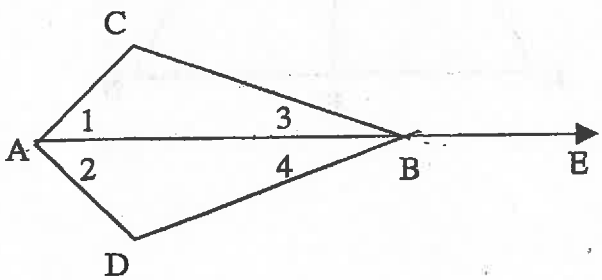
Prove: $\Delta AED \cong \Delta BEC$



Proof Practice

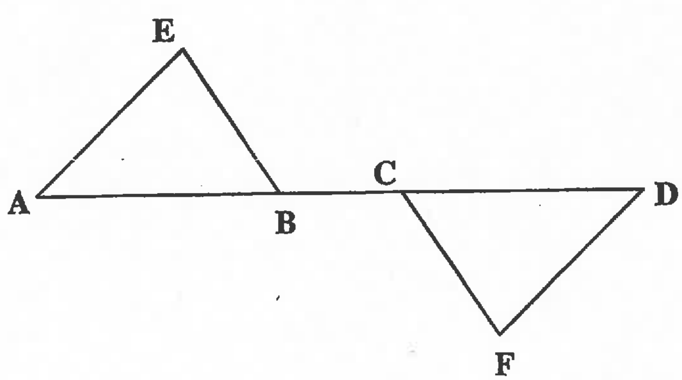
1. Given: $\overrightarrow{ABE}$
   $\overrightarrow{ABE}$ bisects $\angle CAD$
   $\angle CBE \cong \angle DBE$

   Prove: $\Delta ACB \cong \Delta ADB$



2. Given: $\angle A \cong \angle D$
   $\overline{AE} \cong \overline{DF}$
   $\overline{AC} \cong \overline{DB}$

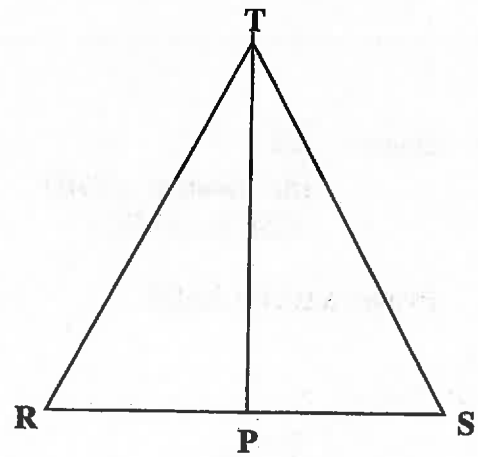
   Prove: $\Delta AEB \cong \Delta DFC$



3. Given: Isosceles ΔRST with $\overline{RT} \cong \overline{ST}$

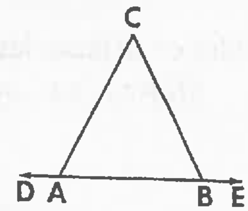
$\overline{TP}$ is a median to base $\overline{RS}$

Prove: $\Delta RTP \cong \Delta STP$

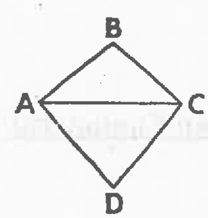


Proofs Involving Isosceles Triangles

Given: $\triangle ABC$ with $\overline{CA} \cong \overline{CB}$ and $\overleftrightarrow{DABE}$.
Prove: $\angle CAD \cong \angle CBE$.



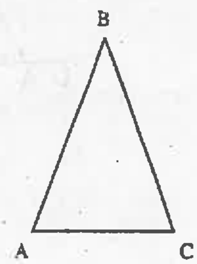
2. *Given:* Isosceles triangles ABC and ADC have the common base $\overline{AC}$.
Prove: $\angle BAD \cong \angle BCD$.



Isosceles and Equilateral Triangles

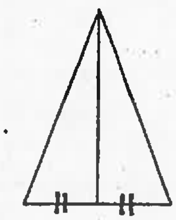
Properties of an Isosceles Δ

1. If two sides of an isosceles Δ are congruent, then the angles opposite those sides are congruent. (Base ∠'s of an isosceles Δ are congruent.)

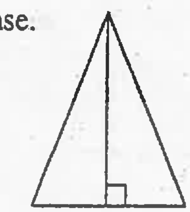


Since $\overline{AB} \cong \overline{BC}$, then $\angle A \cong \angle C$.

2. The bisector of a vertex angle of an isosceles triangle *bisects the base*.

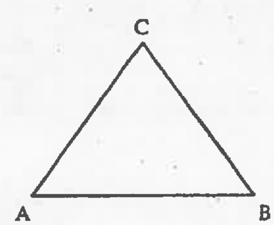


3. The bisector of the vertex angle of an isosceles triangle is *perpendicular* to the base.



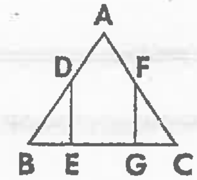
Properties of an Equilateral Triangle

1. Every equilateral triangle is *equiangular*.

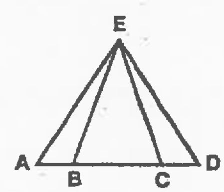


$\angle A \cong \angle B \cong \angle C$

3. In $\triangle ABC$, $\overline{AB} \cong \overline{AC}$, $\overline{DE} \perp \overline{BC}$, $\overline{FG} \perp \overline{BC}$, and $\overline{BG} \cong \overline{CE}$. Prove that $\overline{BD} \cong \overline{CF}$.

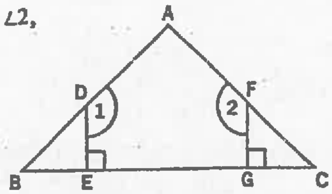


4. *Given:* $\overline{ABCD}$, $\overline{AB} \cong \overline{DC}$, $\angle EBC \cong \angle ECB$.
Prove: $\triangle EAD$ is an isosceles triangle.



More Proofs Involving Isosceles Triangles

1. *Given:* $\overline{BEGC}$, $\overline{BDA}$, $\overline{AFC}$, $\overline{DE} \perp \overline{BC}$, $\overline{FG} \perp \overline{BC}$, $\angle 1 \cong \angle 2$, and $\overline{DE} \cong \overline{FG}$.
Prove: $\triangle ABC$ is isosceles.

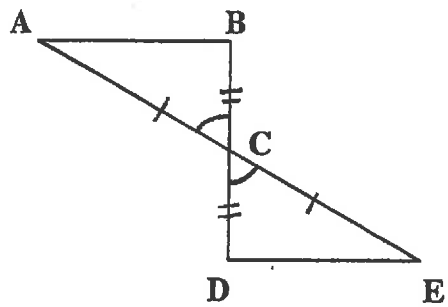


Corresponding Parts of Congruent Triangles

Remember: When any two triangles are congruent, their corresponding sides are congruent and their corresponding angles are congruent.

Examples: A) Name 2 triangles that are congruent
B) State the reason why the triangles are congruent
C) Name 3 additional pairs of congruent parts

1.

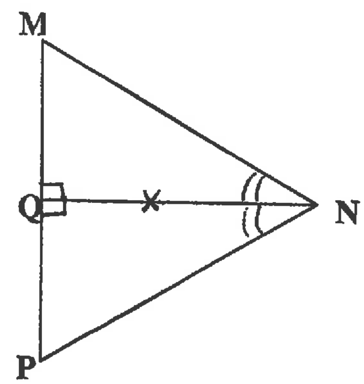


A)________________________

B)________________________

C)________________________

2.



A)________________________

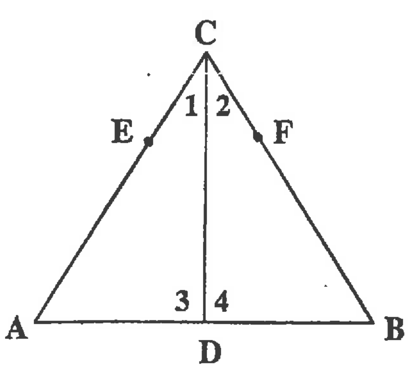
B)________________________

C)________________________

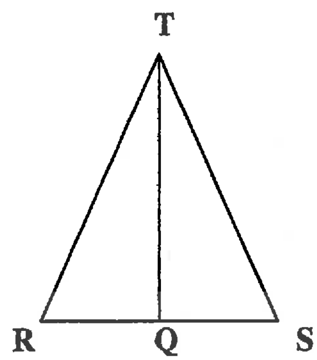
---

3. Given: $\angle 1 \cong \angle 2$
$\overline{CE} \cong \overline{CF}$
$\overline{EA} \cong \overline{FB}$

Prove: $\angle 3 \cong \angle 4$

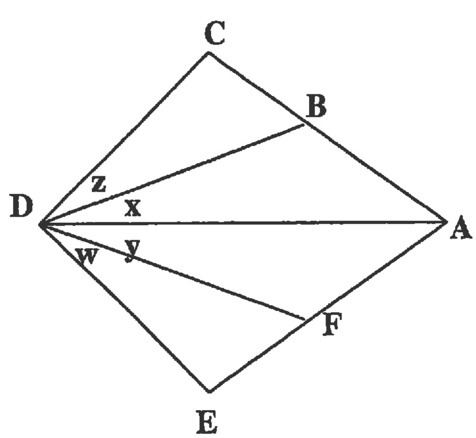


4. If $\overline{TQ}$ bisects $\angle RTS$ and $\overline{TQ} \perp \overline{RS}$, prove that $\overline{TQ}$ bisects $\overline{RS}$.



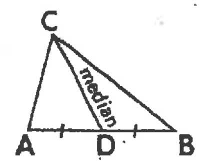
5. Given: $\overline{DC} \cong \overline{DE}$
$\angle x \cong \angle y$
$\angle z \cong \angle w$

Prove: $\overline{AE} \cong \overline{AC}$

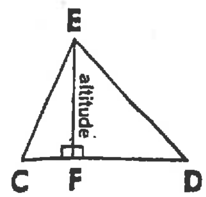


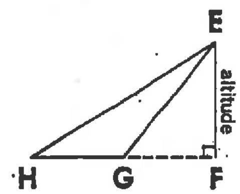
Line Segments Associated With Triangles

Median of a Triangle: A line segment that joins any vertex of the triangle to the *midpoint* of the opposite side.

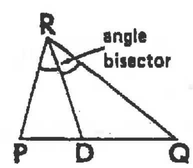


Altitude of a Triangle: A line segment drawn from any vertex of the triangle, *perpendicular* to and ending in the line that contains the opposite side.

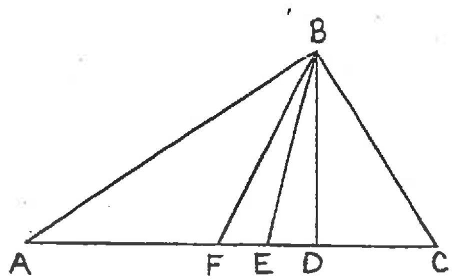




Angle Bisector of a Triangle: A line segment that bisects any angle of the triangle and terminates in the side opposite that angle.



In a scalene triangle, the *altitude*, the *median*, and the *angle bisector* drawn from any common vertex are three distinct line segments.

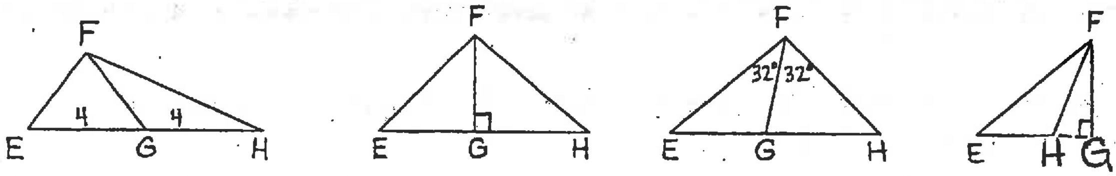


$\overline{BD}$ is the *altitude* from *B* because $\overline{BD} \perp \overline{AC}$;

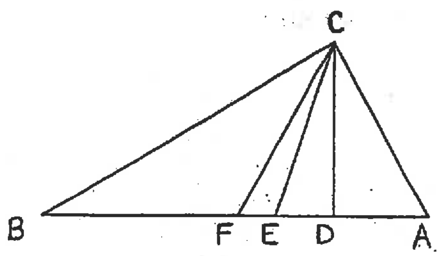
$\overline{BE}$ is the *angle bisector* from *B* because $\angle ABE \cong \angle EBC$;

$\overline{BF}$ is the *median* from *B* because *F* is the midpoint of $\overline{AC}$.

Name the type of line segment that $\overline{FG}$ is in each triangle.



Polygon ABC is a triangle. $\overline{CD}$ is an altitude. $\overline{CE}$ is an angle bisector. $\overline{CF}$ is a median.



a. Name two line segments which are congruent. ________________

b. Name two angles which are right angles. ________________

c. Name two congruent angles, each of which has its vertex at C. ________________

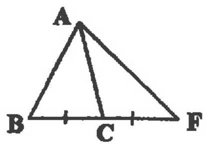
d. Name two line segments which are perpendicular to each other. ________________

Worksheet Altitude, Median,
Angle bisector, perpendicular Bisector

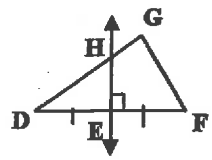
Name ____________________

Name the special segment for 1-4

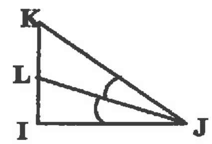
1) $\overline{AC}$



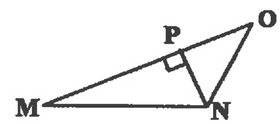
2) $\overleftrightarrow{HE}$



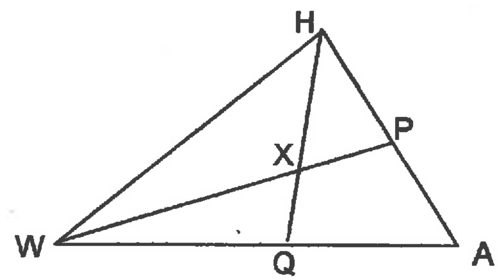
3) $\overline{JL}$



4) $\overline{PN}$



In $\triangle$ AHW, m∠A = 64 and m∠AWH = 36. If $\overline{WP}$ is an angle bisector and $\overline{HQ}$ is an altitude, find each measure.



5. m∠AQH = ____________

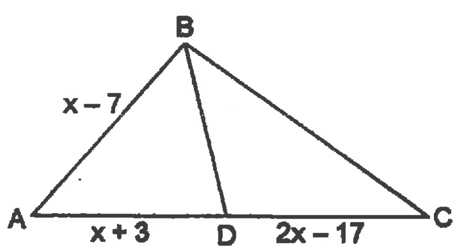
6. m∠APW = ____________

7. m∠AHQ = ____________

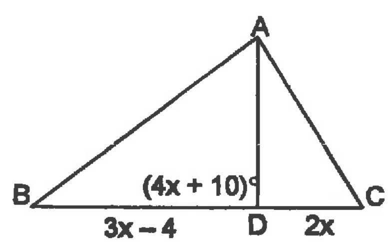
8. m∠HXW = ____________

9. If $\overline{WP}$ is a median, AP = 3y + 11 and PH = 7y – 5, find AH.

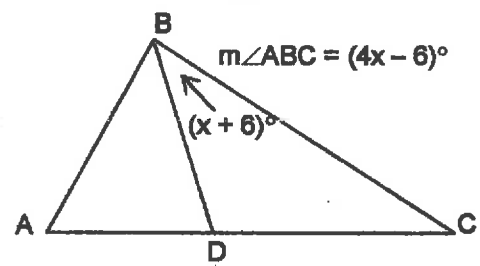
10). Find $\overline{AB}$ if $\overline{BD}$ is a median of $\triangle$ABC.



11. Find $\overline{BC}$ if AD is an altitude of $\triangle$ ABC.



12. Find m∠ABC if $\overline{BD}$ is an angle bisector of $\triangle$ ABC.



Overlapping Triangles

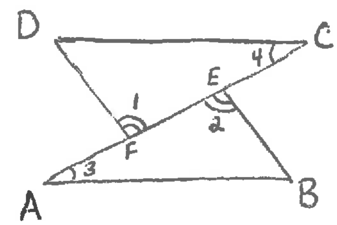
1. ***Given:*** $\overline{AFEC}$

   $\overline{AF} \cong \overline{EC}$

   $< 3 \cong < 4$

   $< 1 \cong < 2$

   Prove: $\Delta ABE \cong \Delta CDF$



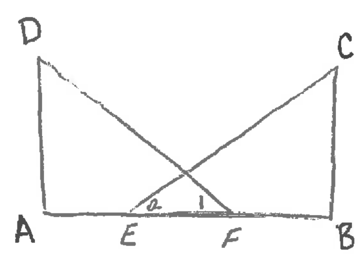
2. ***Given:*** $\overline{AFEB}$

   $\overline{CE} \cong \overline{DF}$

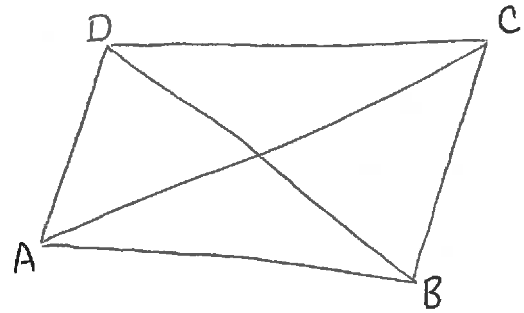
   $< 1 \cong < 2$

   $\overline{AE} \cong \overline{BF}$

   Prove: $\Delta AFD \cong \Delta BEC$

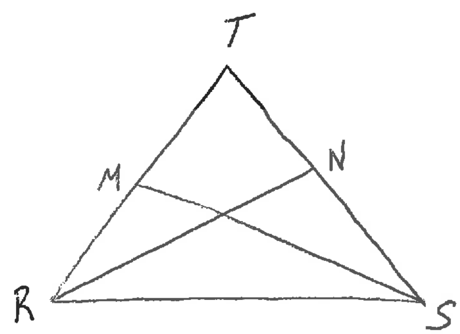


3. ***Given:*** $\overline{DA} \cong \overline{CB}$
$\overline{DA} \perp \overline{AB}$
$\overline{CA} \perp \overline{AB}$



Prove: $\Delta DAB \cong \Delta CBA$

4. ***Given:*** $\overline{TR} \cong \overline{TS}$
$\overline{MR} \cong \overline{NS}$

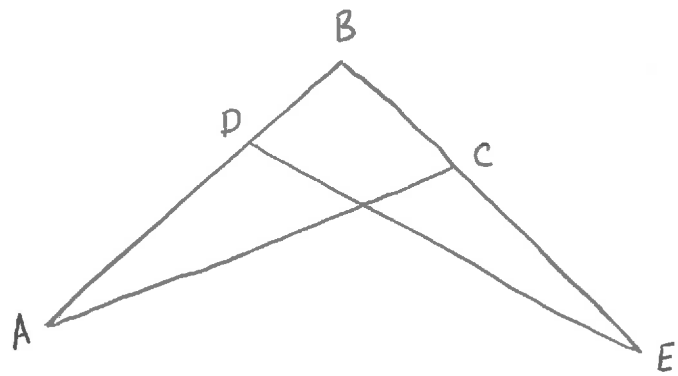


Prove: $\Delta RTN \cong \Delta STM$

Overlapping Triangle Proofs

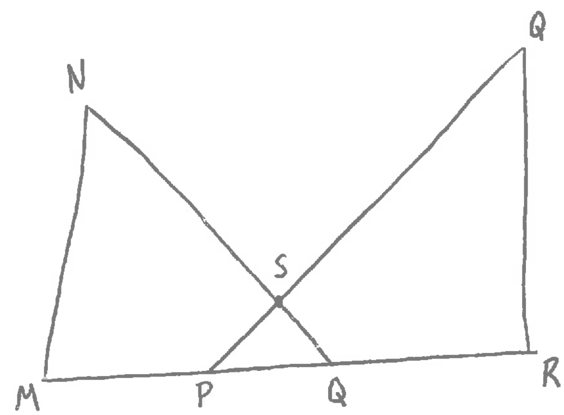
1. ***Given :*** $\overline{BD} \cong \overline{BC}$
   $\overline{DA} \cong \overline{CE}$

   Prove: $< A \cong < E$



2. ***Given :*** $\overline{NM} \cong \overline{QR}$
   $\overline{NM} \perp \overline{MR}, \overline{QR} \perp \overline{MR}$

   Prove: $< N \cong < Q$



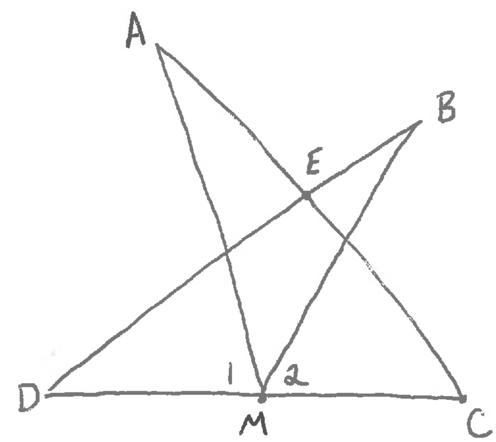
3. ***Given***: $\overline{AC}$ *and* $\overline{BD}$ intersect at E

$< D \cong < C$

M is the midpoint of $\overline{DC}$

$< 1 \cong < 2$

Prove: $\overline{DB} \cong \overline{CA}$



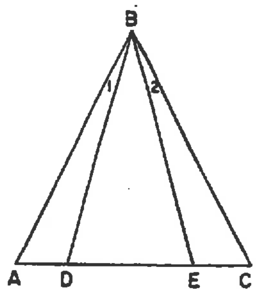
Math 2R

Homework – More Proofs Involving Isosceles Triangles & Corresponding Parts

Name ________________________

Date ________________________

1. Given: $\triangle ABC$, $\overline{BD} \cong \overline{BE}$, $\overline{ADEC}$, and $\angle 1 \cong \angle 2$.



Prove: $\triangle ABC$ is isosceles.

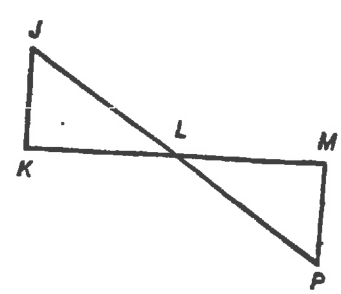
2. Given: $\overline{JK} \cong \overline{PM}$

$\overline{JK} \perp \overline{KM}$

$\overline{PM} \perp \overline{KM}$

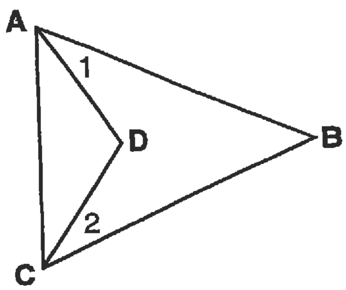
$\angle J \cong \angle P$

Prove: $\overline{KM}$ bisects $\overline{JP}$



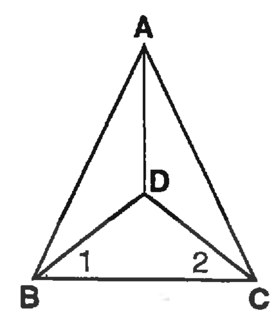
3. Given: $\overline{AB} \cong \overline{BC}$
$\angle 1 \cong \angle 2$

Prove: $\triangle ADC$ is isosceles



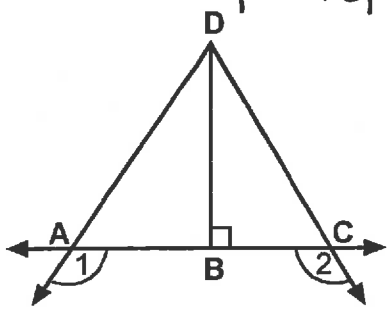
4. Given: $\overline{AB} \cong \overline{AC}$
$\overline{AD}$ bisects $\angle BAC$

Prove: $\angle 1 \cong \angle 2$



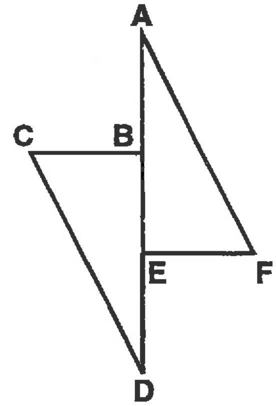
Name: ______________________________

) Given: $\angle 1 \cong \angle 2$ and $\overline{DB} \perp \overline{AC}$
B is the midpoint of $\overline{AC}$



Prove by any method: $\triangle ABD \cong \triangle CBD$

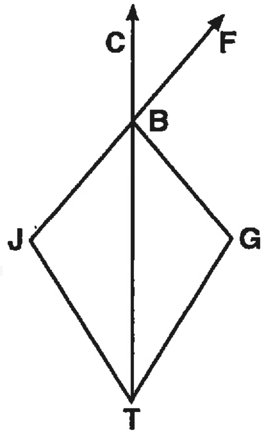
2)



Given: $\overline{AB} \cong \overline{ED}$
$\overline{FE} \cong \overline{CB}$
$\overline{FE} \perp \overline{AD}$
$\overline{CB} \perp \overline{AD}$

Prove by any method: $\triangle AEF \cong \triangle CBD$

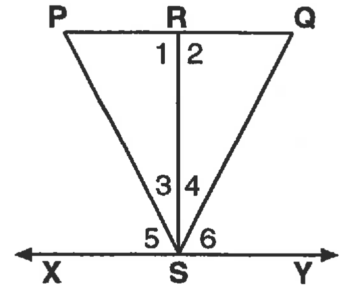
3)



Given: ∠CBF ≅ ∠TBG
$\overline{TB}$ bisects ∠JTG

Prove by any method: ΔBJT ≅ ΔBGT

4)



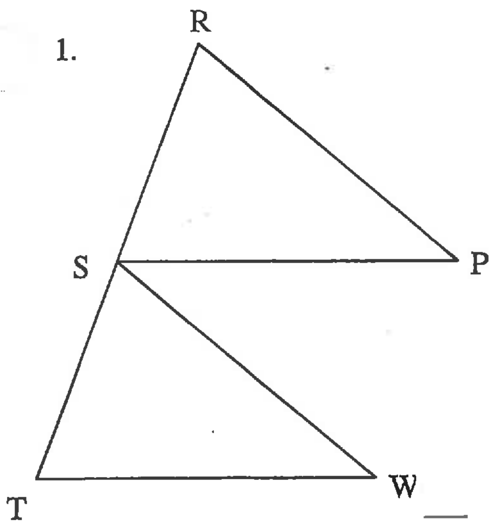
Given: $\overline{RS} \perp \overline{XY}$
∠5 ≅ ∠6
∠P ≅ ∠5
∠Q ≅ ∠6
$\overline{PS} \cong \overline{QS}$

Prove by any method: ΔPRS ≅ ΔQRS

Name:____________________________________
Date:_____________________________________

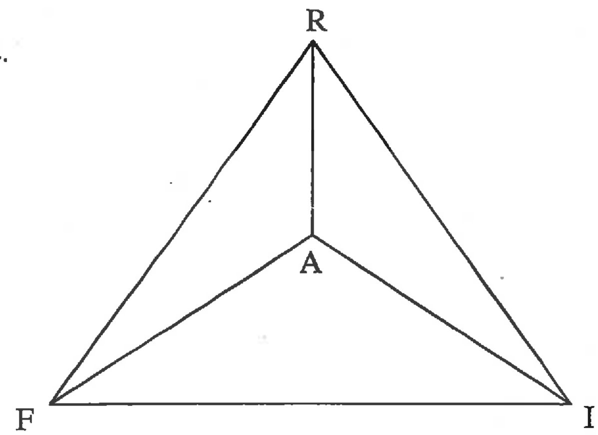
Geometry R
Practice Test 1

1.



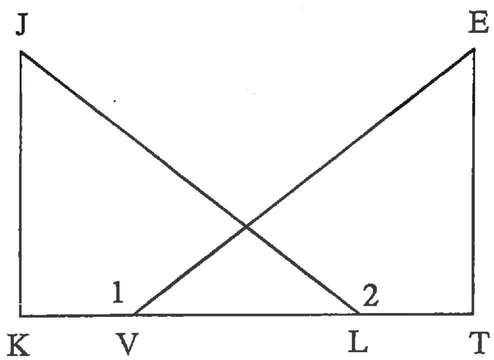
Given: S is the midpoint of $\overline{RT}$
$\overline{SW} \cong \overline{SP}$
$\angle RSW \cong \angle TSP$
Prove: $\Delta TSW \cong \Delta SRP$

2.



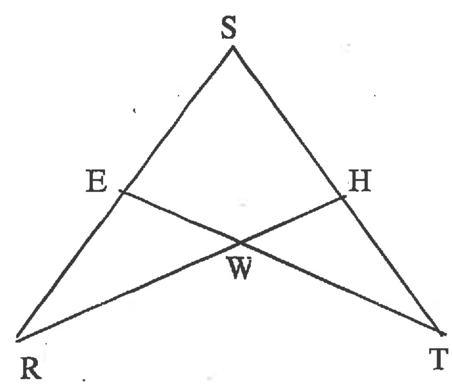
Given: $\overline{AR}$ bisects $\angle FAI$
$\overline{FA} \cong \overline{IA}$
Prove: ΔFRI is isosceles

3.



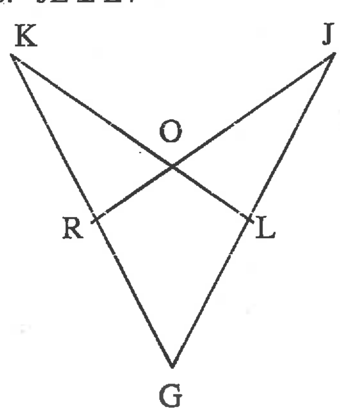
Given: $\overline{JK} \perp \overline{KT}$, $\overline{ET} \perp \overline{KT}$
$\angle 1 \cong \angle 2$, $\overline{KV} \cong \overline{TL}$
Prove: $\overline{JL} \cong \overline{EV}$

4.



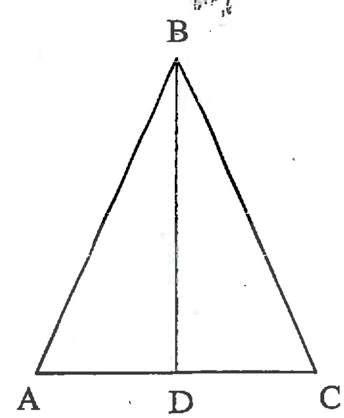
Given: $\angle R \cong \angle T$
$\overline{SR} \cong \overline{ST}$
Prove: $\angle SET \cong \angle SHR$

5.



Given: $\overline{JR} \perp \overline{KG}$, $\overline{KL} \perp \overline{JG}$
$\overline{RO} \cong \overline{LO}$
Prove: $\Delta KOR \cong \Delta JOL$

6.



Given: $\overline{BD}$ is the altitude to $\overline{AC}$
$\overline{BD}$ bisects $\angle ABC$
Prove: $\overline{AD} \cong \overline{CD}$