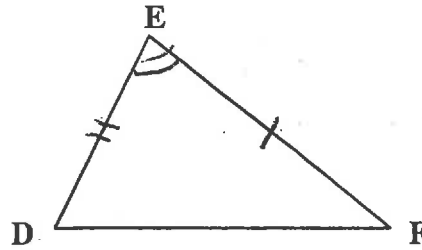
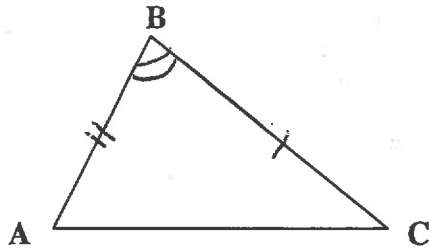


# SAS Postulate

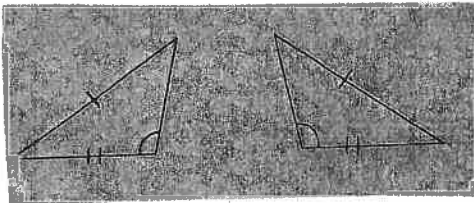
Two triangles are congruent if **two sides and the included angle** of one triangle are congruent respectively to two sides and the included angle of the other triangle.



Note: The angle must be included between the two congruent sides!!

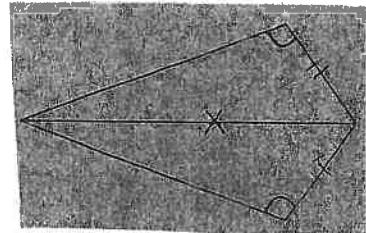
Examples: Is the given information sufficient to prove the triangles are congruent?

1.



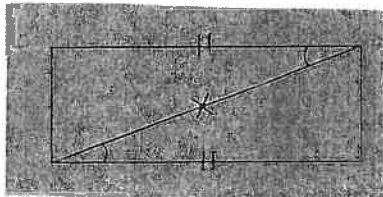
Not SAS

2.



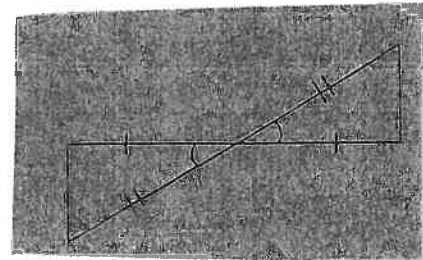
Not SAS

3.



YES SAS

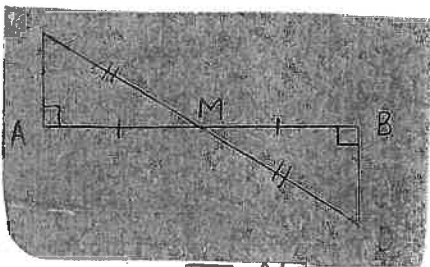
4.



yes SAS

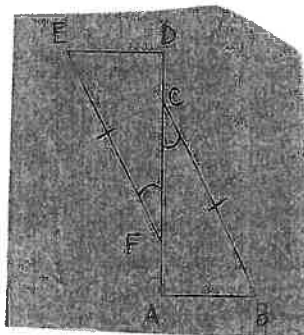
Name the pair of corresponding sides or pair of corresponding angles that would be needed in order to prove the triangles are congruent by SAS.

5.



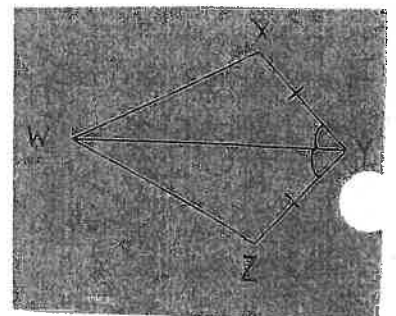
either  $\overline{CA} \cong \overline{DB}$   
OR  $\angle CMA \cong \angle DMB$

6.



$\overline{FD} \cong \overline{CA}$

7.



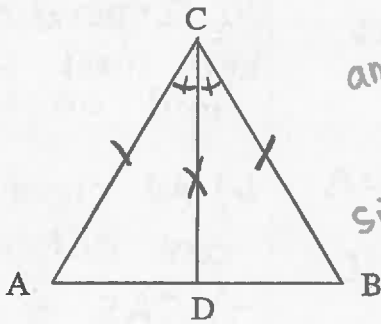
$\overline{WY} \cong \overline{WY}$



## Proofs and the SAS Postulate

1. Given:  $\overline{AC} \cong \overline{BC}$   
 $\overline{CD}$  bisects  $\angle ACB$

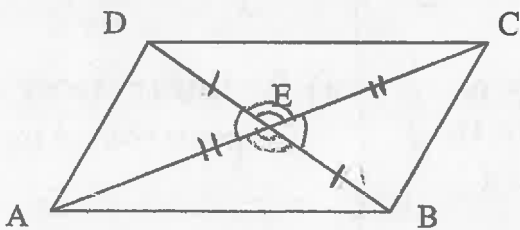
Prove:  $\triangle ACD \cong \triangle BCD$



Statements	Reasons
side 1) $\overline{AC} \cong \overline{BC}$	1) Given
angle 2) $\overline{CD}$ bisects $\angle ACB$	2) Given
angle 3) $\angle ACD \cong \angle BCD$	3) A bisector divides a segment into 2 $\cong$ segments
side 4) $\overline{CD} \cong \overline{CD}$	4) Reflexive property
5) $\triangle ACD \cong \triangle BCD$	5) SAS $\cong$ SAS

2. Given:  $\overline{DB}$  and  $\overline{AC}$  bisect each other at E

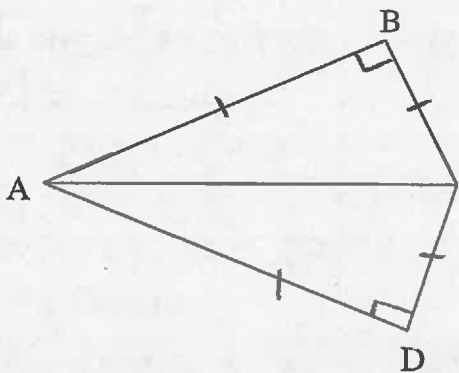
Prove:  $\triangle AEB \cong \triangle CED$



Statements	Reasons
1) $\overline{DB}$ and $\overline{AC}$ bisect each other at E	1) Given
sides 2) $\overline{DE} \cong \overline{EB}$ $\overline{AE} \cong \overline{EC}$	2) A bisector divides a segment into 2 congruent segments
angle 3) $\angle DEC \cong \angle AEB$	3) All vertical angles are $\cong$
4) $\triangle AEB \cong \triangle CED$	4) SAS $\cong$ SAS

3. Given:  $\overline{AB} \perp \overline{BC}$   
 $\overline{AD} \perp \overline{DC}$   
 $\overline{AB} \cong \overline{AD}$   
 $\overline{BC} \cong \overline{DC}$

Prove:  $\triangle ABC \cong \triangle ADC$



- 1)  $\overline{AB} \perp \overline{BC}$   
 2)  $\overline{AD} \perp \overline{DC}$   
 side 3)  $\overline{AB} \cong \overline{AD}$   
 side 4)  $\overline{BC} \cong \overline{DC}$

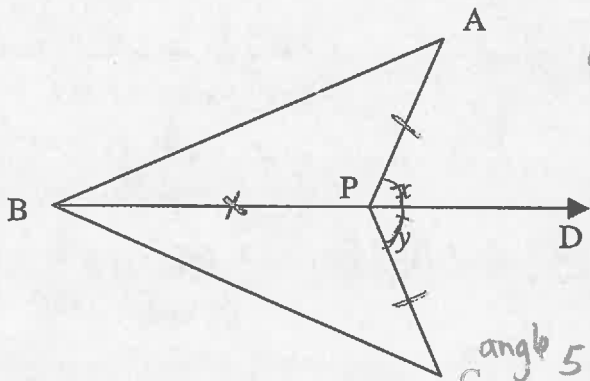
5)  $\angle ABC$  and  $\angle CDA$  are right angles

- angle C 6)  $\angle ABC \cong \angle CDA$   
 7)  $\triangle ABC \cong \triangle ADC$

- 1) given  
 2) given  
 3) given  
 4) given  
 5) Perpendicular lines meet to form right angles  
 6) All right angles are congruent  
 7) SAS  $\cong$  SAS

4. Given:  $\overline{AP} \cong \overline{CP}$   
 $\angle x \cong \angle y$   
 $\overline{BD}$  is a straight line

Prove:  $\triangle BAP \cong \triangle BCP$



- statements  
 side 1)  $\overline{AP} \cong \overline{CP}$

- angle 2)  $\angle x \cong \angle y$   
 3)  $\overline{BD}$  is a straight line

- 4)  $\angle APB$  is a supplementary angle to  $\angle x$   
 $\angle CPB$  is a supplementary angle to  $\angle y$

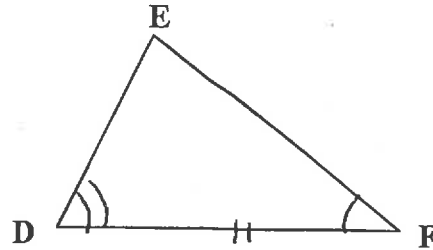
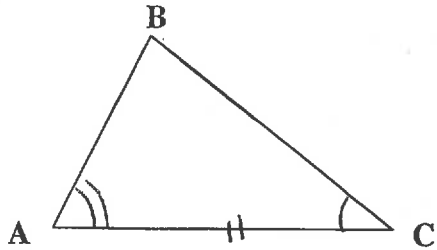
- angle 5)  $\angle APB \cong \angle CPB$   
 side 6)  $\overline{BP} \cong \overline{BP}$

- 7)  $\triangle BAP \cong \triangle BCP$

- Reasons  
 1) given  
 2) given  
 3) given  
 4) A linear pair form supplementary angles  
 5) Supplements of  $\cong$  angles are  $\cong$   
 6) Reflexive  
 7) SAS  $\cong$  SAS

# ASA Postulate

Two triangles are congruent if **two angles and the included side** of one triangle are congruent respectively to two angles and the included side of the other triangle.



Note: The side must be included between the two congruent angles!!

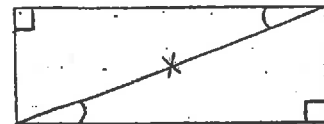
Examples: Is the given information sufficient to prove the triangles are congruent by ASA?

1.



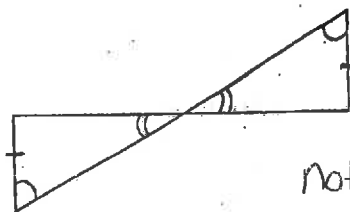
Yes ASA

2.



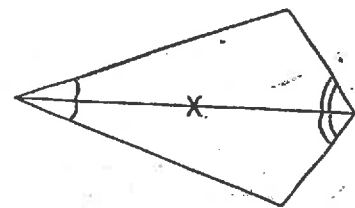
Not ASA

3.



Not ASA

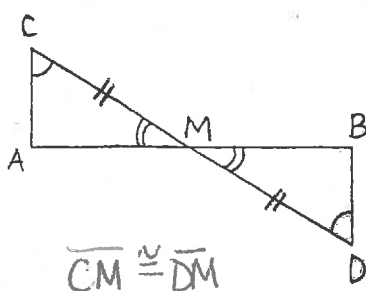
4.



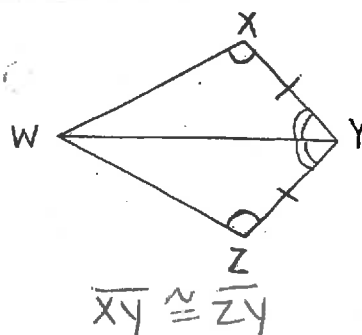
Yes ASA

Name the pair of corresponding sides or pair of corresponding angles that would be needed in order to prove the triangles are congruent by ASA.

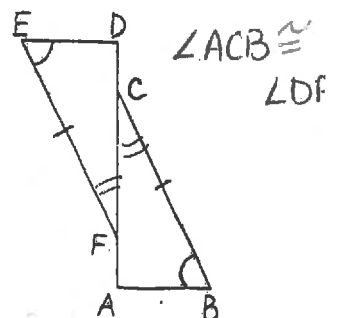
5.



6.



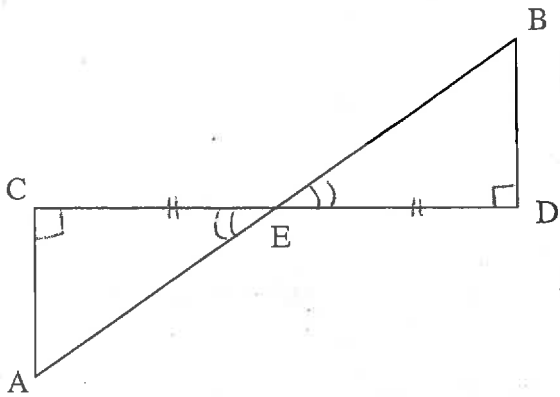
7.



Proofs:

1. Given:  $\overline{BA}$  bisects  $\overline{CD}$   
 $\overline{AC} \perp \overline{CD}$   
 $\overline{BD} \perp \overline{CD}$

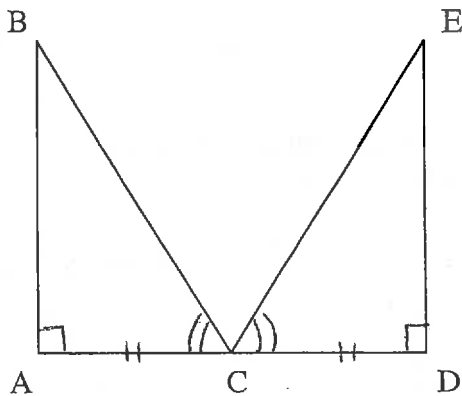
Prove:  $\triangle ACE \cong \triangle BDE$



Statements	Reasons
1. $\overline{BA}$ bisects $\overline{CD}$	1. Given
2. $\overline{CE} \cong \overline{DE}$	2. A bisector cuts a segment into 2 $\cong$ segments.
3. $\overline{AC} \perp \overline{CD}$ , $\overline{BD} \perp \overline{CD}$	3. Given
4. $\angle C$ & $\angle D$ are right $\angle$ 's	4. $\perp$ lines meet to form right $\angle$ 's
5. $\angle C \cong \angle D$	5. All right $\angle$ 's are $\cong$ .
6. $\angle CEA \cong \angle DEB$	6. Vertical $\angle$ 's are $\cong$
7. $\triangle ACE \cong \triangle BDE$	7. ASA $\cong$ ASA

2. Given: C is the midpoint of  $\overline{AD}$   
 $\overline{BA} \perp \overline{AD}$ ,  $\overline{ED} \perp \overline{DA}$   
 $\angle BCA \cong \angle ECD$

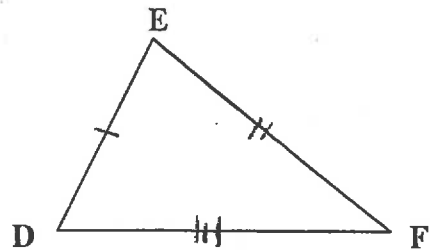
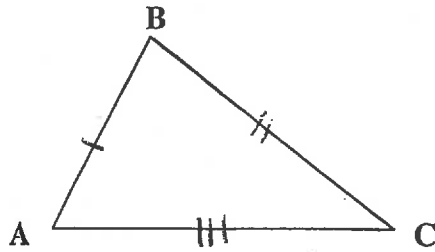
Prove:  $\triangle BAC \cong \triangle EDC$



Statements	Reasons
1. C is the midpoint of $\overline{AD}$	1. Given
2. $\overline{AC} \cong \overline{CD}$	2. A mid point cuts a segment into 2 $\cong$ segments.
3. $\angle BCA \cong \angle ECD$	3. Given
4. $\overline{BA} \perp \overline{AD}$ , $\overline{ED} \perp \overline{DA}$	4. Given
5. $\angle A$ & $\angle D$ are right $\angle$ 's	5. $\perp$ lines meet to form right $\angle$ 's
6. $\angle A \cong \angle D$	6. All right $\angle$ 's are $\cong$
7. $\triangle BAC \cong \triangle EDC$	7. ASA $\cong$ ASA

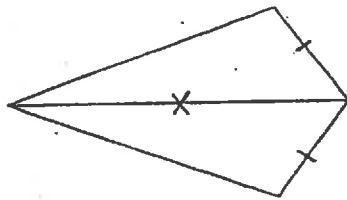
# SSS Postulate

Two triangles are congruent if **the three sides** of one triangle are congruent respectively to the three sides of the other triangle.



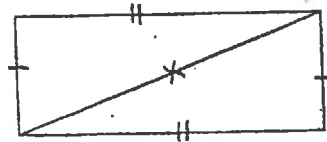
Examples: Is the given information sufficient to prove the triangles are congruent by SSS?

1.



Not SSS

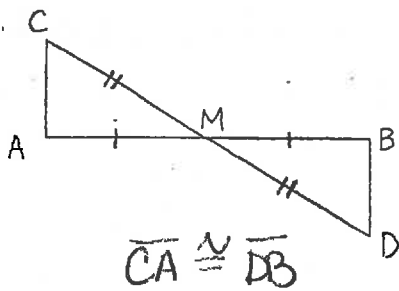
2.



Yes SSS

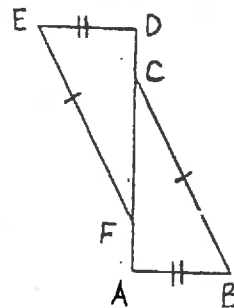
Name the pair of corresponding sides or pair of corresponding angles that would be needed in order to prove the triangles are congruent by ASA.

3.



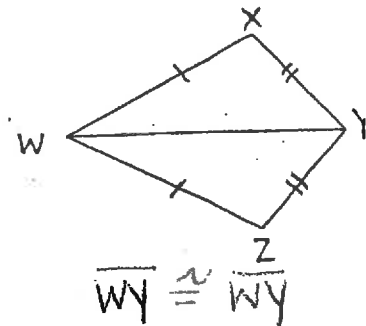
$\overline{CA} \cong \overline{DB}$

4.



$\overline{DE} \cong \overline{AC}$

5.

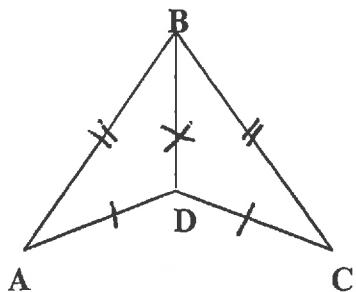


$\overline{WY} \cong \overline{ZY}$

Proofs:

1. Given:  $\overline{AB} \cong \overline{CB}$   
 $\overline{AD} \cong \overline{CD}$

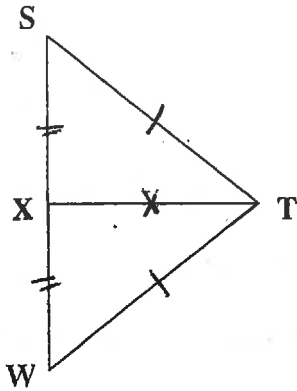
Prove:  $\triangle ABD \cong \triangle CBD$



Statements	Reasons
① 1. $\overline{AB} \cong \overline{CB}$	1. Given
② 2. $\overline{AD} \cong \overline{CD}$	2. Given
③ 3. $\overline{BD} \cong \overline{BD}$	3. Reflexive Postulate
④ 4. $\triangle ABD \cong \triangle CBD$	4. SSS $\cong$ SSS

2. Given:  $x$  is the midpoint of  $\overline{SW}$   
 $\overline{ST} \cong \overline{WT}$

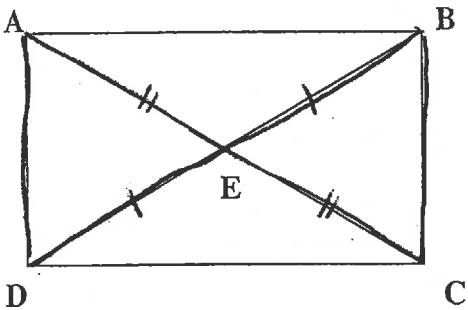
Prove:  $\triangle STX \cong \triangle WTX$



Statements	Reasons
1. $\overline{ST} \cong \overline{WT}$	1. Given
2. $x$ is the midpt of $\overline{SW}$	2. Given
3. $\overline{SX} \cong \overline{WX}$	3. A midpoint cuts a segment into 2 $\cong$ segments.
4. $\overline{XT} \cong \overline{XT}$	4. Reflexive Postulat
5. $\triangle SXT \cong \triangle WXT$	5. SSS $\cong$ SSS

3. Given:  $\overline{AC}$  and  $\overline{BD}$  bisect each other at  $E$   
 $\overline{AD} \cong \overline{BC}$

Prove:  $\triangle AED \cong \triangle BEC$



Statements	Reasons
1. $\overline{AC}$ and $\overline{BD}$ bisect each other at $E$	1. Given
2. $\overline{AE} \cong \overline{CE}$ $\overline{BE} \cong \overline{DE}$	2. A bisector cuts a segment into 2 $\cong$ segments
3. $\overline{AD} \cong \overline{BC}$	3. Given
4. $\triangle AED \cong \triangle BEC$	4. SSS $\cong$ SSS

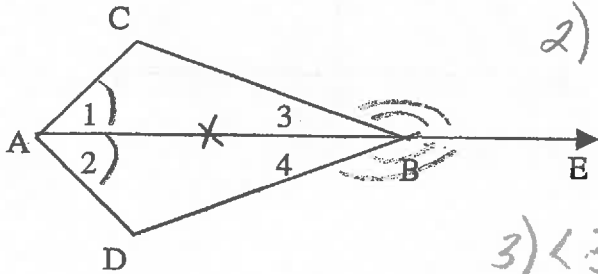
\* Note: You could also do this proof by SAS if you used vertical  $\angle$ 's.



# Proof Practice

1. Given:  $\overline{ABE}$   
 $\overline{ABE}$  bisects  $\angle CAD$   
 $\angle CBE \cong \angle DBE$

Prove:  $\triangle ACB \cong \triangle ADB$



2)  $\angle 1 \cong \angle 2$

3)  $\angle 3$  and  $\angle CBE$   
are supp  
 $\angle 4$  and  $\angle DBE$   
are supp

4)  $\angle 3 \cong \angle 4$

5)  $\overline{AB} \cong \overline{AB}$

6)  $\triangle ACB \cong \triangle ADB$

Reasons

1) given

2) A bisector cuts a segment into 2  $\cong$  segments

3) If 2  $\angle$ s form a linear pair they are supp

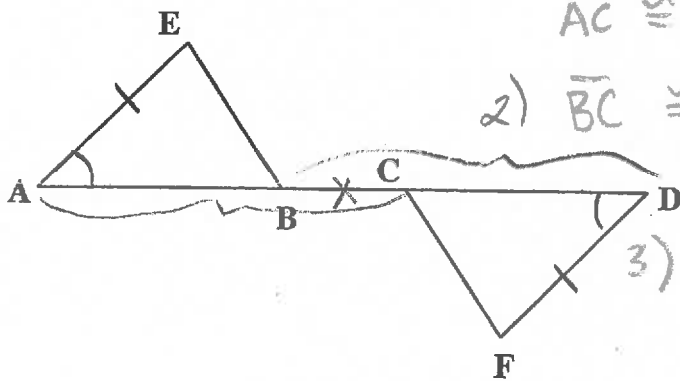
4) Supplements of  $\cong \angle$ s are  $\cong$

5) Reflexive

6) ASA  $\cong$  ASA

2. Given:  $\angle A \cong \angle D$   
 $\overline{AE} \cong \overline{DF}$   
 $\overline{AC} \cong \overline{DB}$

Prove:  $\triangle AEB \cong \triangle DFC$



1)  $\angle A \cong \angle D, \overline{AE} \cong \overline{DF}$   
 $\overline{AC} \cong \overline{DB}$

2)  $\overline{BC} \cong \overline{BC}$

3)  $\overline{AB} \cong \overline{CD}$

4)  $\triangle AEB \cong \triangle DFC$

Reasons

1) given

2) Reflexive

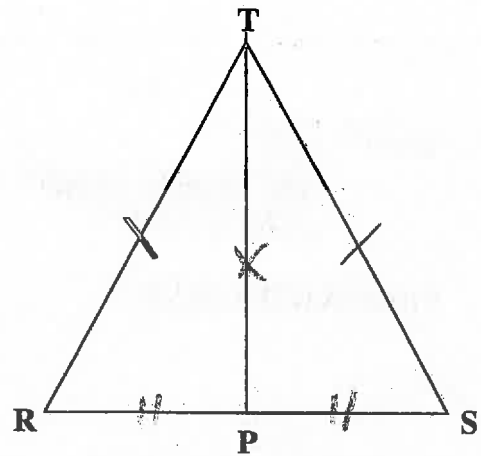
3) subtraction

4) SAS  $\cong$  SAS

3. Given: Isosceles  $\triangle RST$  with  $\overline{RT} \cong \overline{ST}$

$\overline{TP}$  is a median to base  $\overline{RS}$

Prove:  $\triangle RTP \cong \triangle STP$

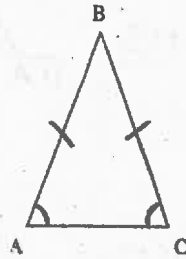


Statements	Reasons
1) $\overline{RT} \cong \overline{ST}$ $\overline{TP}$ median to base $\overline{RS}$	1) given
2) P is the midpt of $\overline{RS}$	2) Median gives us midpt
3) $\overline{RP} \cong \overline{PS}$	3) A midpt divides a segment into 2 $\cong$ segments
4) $\overline{TP} \cong \overline{TP}$	4) Reflexive
5) $\triangle RTP \cong \triangle STP$	5) SSS $\cong$ SSS

# Isosceles and Equilateral Triangles

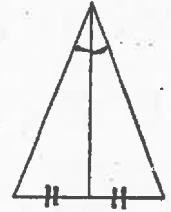
## Properties of an Isosceles $\Delta$

1. If two sides of an isosceles  $\Delta$  are congruent, then the angles opposite those sides are congruent.  
(Base  $\angle$ 's of an isosceles  $\Delta$  are congruent.)

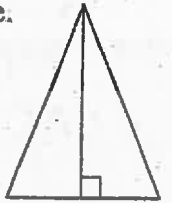


Since  $\overline{AB} \cong \overline{BC}$ , then  $\angle A \cong \angle C$ .

2. The bisector of a vertex angle of an isosceles triangle *bisects the base*.



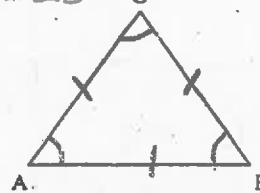
3. The bisector of the vertex angle of an isosceles triangle is *perpendicular* to the base.



\* For any isosceles  $\Delta$ , the altitude, the median, and the angle bisector drawn from the vertex angle to the opposite side are the same line segment. This line segment separates the  $\Delta$  into 2  $\cong$   $\Delta$ s.

## Properties of an Equilateral Triangle

1. Every equilateral triangle is *equiangular*.



$$\angle A \cong \angle B \cong \angle C$$

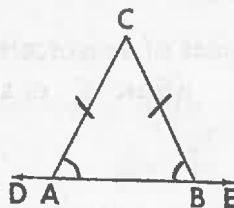
Proof: If  $\Delta ABC$  is equilateral, then  $\overline{AB} \cong \overline{BC} \cong \overline{CA}$

By isosceles  $\Delta$  thm, since  $\overline{AB} \cong \overline{BC}$ ,  $\angle A \cong \angle C$   
and since  $\overline{BC} \cong \overline{CA}$ ,  $\angle B \cong \angle A$

Therefore  $\angle A \cong \angle B \cong \angle C$  (Transitive)

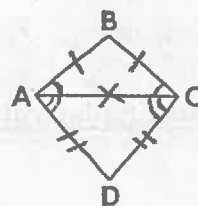
## Proofs Involving Isosceles Triangles

Given:  $\triangle ABC$  with  $\overline{CA} \cong \overline{CB}$  and  $\overleftrightarrow{DABE}$   
 Prove:  $\angle CAD \cong \angle CBE$ .



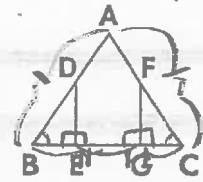
Statements	Reasons
1) $\triangle ABC$ , $\overline{CA} \cong \overline{CB}$ and $\overleftrightarrow{DABE}$	1) given
2) $\triangle ABC$ is an isosceles $\triangle$	2) An isosceles $\triangle$ has 2 $\cong$ sides.
3) $\angle CAB \cong \angle CBA$	3) Base angles of an isosceles $\triangle$ are congruent.
4) $\angle CAB$ and $\angle CAD$ are supplementary and $\angle CBA$ and $\angle CBE$ are supplementary	4) If 2 angles form a linear pair, then they are supplementary
5) $\angle CAD \cong \angle CBE$	5) Supplements of congruent angles are congruent.

2. Given: Isosceles triangles  $ABC$  and  $ADC$  have the common base  $\overline{AC}$   
 Prove:  $\angle BAD \cong \angle BCD$ .



Statements	Reasons
1) $\triangle ABC$ and $\triangle ADC$ are isosceles $\triangle$ s and have a common base $\overline{AC}$	1) Given
2) $\angle BAC \cong \angle BCA$ and $\angle DAC \cong \angle DCA$	2) Base angles of an isosceles $\triangle$ are congruent
3) $\angle BAC + \angle DAC \cong \angle BCA + \angle DCA$	3) Addition
4) $\angle BAD = \angle BAC + \angle DAC$ $\angle BCD = \angle BCA + \angle DCA$	4) Partition
5) $\angle BAD \cong \angle BCD$	5) Substitution

3. In  $\triangle ABC$ ,  $\overline{AB} \cong \overline{AC}$ ,  $\overline{DE} \perp \overline{BC}$ ,  $\overline{FG} \perp \overline{BC}$ , and  $\overline{BG} \cong \overline{CE}$ . Prove that  $\overline{BD} \cong \overline{CF}$ .



Statements

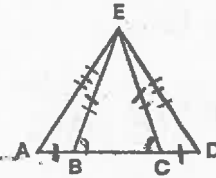
Reasons

- |   |   |
|---|---|
| <p>1) <math>\overline{AB} \cong \overline{AC}</math>, <math>\overline{BG} \cong \overline{CE}</math><br/><math>\overline{DE} \perp \overline{BC}</math>, <math>\overline{FG} \perp \overline{BC}</math></p> <p>2) <math>\triangle ABC</math> is an isosceles <math>\triangle</math></p> <p>3) <math>\angle B \cong \angle C</math></p> <p>4) <math>\angle DEB</math> and <math>\angle FGC</math> are right angles</p> <p>5) <math>\angle DEB \cong \angle FGC</math></p> <p>6) <math>\overline{EG} \cong \overline{EG}</math></p> <p>7) <math>\overline{BG} - \overline{EG} = \overline{CE} - \overline{EG}</math></p> <p>8) <math>\overline{BE} = \overline{BG} - \overline{EG}</math> and <math>\overline{GC} = \overline{CE} - \overline{EG}</math></p> <p>9) <math>\overline{BE} \cong \overline{GC}</math></p> <p>10) <math>\triangle DEB \cong \triangle FGC</math></p> <p>11) <math>\overline{BD} \cong \overline{CF}</math></p> | <p>1) given</p> <p>2) An isosceles <math>\triangle</math> has 2 <math>\cong</math> sides</p> <p>3) Base angles of an isosceles <math>\triangle</math> are <math>\cong</math></p> <p>4) <math>\perp</math> lines meet to form right angles</p> <p>5) All right angles are <math>\cong</math></p> <p>6) Reflexive</p> <p>7) Subtraction</p> <p>8) Partition</p> <p>9) substitution</p> <p>10) ASA <math>\cong</math> ASA</p> <p>11) CPCTC</p> |
|---|---|

4. Given:  $\overline{ABCD}$ ,  $\overline{AB} \cong \overline{DC}$ ,  $\angle EBC \cong \angle ECB$ .  
Prove:  $\triangle EAD$  is an isosceles triangle.

Statements

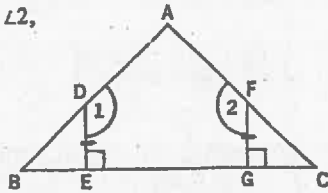
Reasons



- |  |  |
|--|--|
| <p>1) <math>\overline{ABCD}</math>, <math>\overline{AB} \cong \overline{DC}</math>,<br/><math>\angle EBC \cong \angle ECB</math></p> <p>2) <math>\overline{EB} \cong \overline{EC}</math></p> <p>3) <math>\angle EBC</math> and <math>\angle EBA</math> are supp<br/><math>\angle ECB</math> and <math>\angle ECD</math> are supp</p> <p>4) <math>\angle EBA \cong \angle ECD</math></p> <p>5) <math>\triangle AEB \cong \triangle DEC</math></p> <p>6) <math>\overline{AE} \cong \overline{DE}</math></p> <p>7) <math>\triangle AED</math> is isosceles</p> | <p>1) given</p> <p>2) If 2 angles of a <math>\triangle</math> are <math>\cong</math> the sides opposite those angles are <math>\cong</math></p> <p>3) If 2 angles for a linear pair, then they are supplementary</p> <p>4) Supplements of <math>\cong</math> angles are <math>\cong</math></p> <p>5) SAS <math>\cong</math> SAS</p> <p>6) CPCTC</p> <p>7) An isosceles <math>\triangle</math> has 2 <math>\cong</math> sides</p> |
|--|--|

## More Proofs Involving Isosceles Triangles

1. Given:  $\overline{BE} \perp \overline{GC}$ ,  $\overline{BDA}$ ,  $\overline{AFC}$ ,  $\overline{DE} \perp \overline{BC}$ ,  $\overline{FG} \perp \overline{BC}$ ,  $\angle 1 \cong \angle 2$ ,  
and  $\overline{DE} \cong \overline{FG}$ .  
Prove:  $\triangle ABC$  is isosceles.

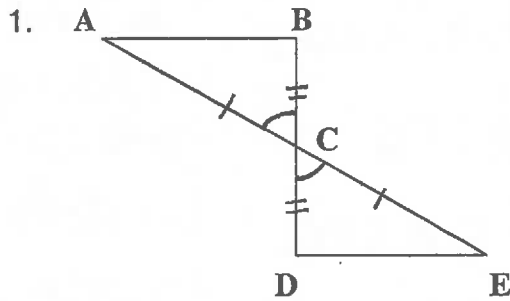


Statements	Reasons
1) $\overline{BE} \perp \overline{GC}$ , $\overline{BDA}$ , $\overline{AFC}$ , $\overline{DE} \perp \overline{BC}$ , $\overline{FG} \perp \overline{BC}$ , $\angle 1 \cong \angle 2$ , $\overline{DE} \cong \overline{FG}$	1) Given
2) $\angle DEB$ and $\angle FGC$ are right angles	2) $\perp$ lines meet to form right angle
3) $\angle DEB \cong \angle FGC$	3) All right angles are $\cong$ .
4) $\angle 1$ and $\angle EDB$ are supplementary $\angle 2$ and $\angle GFC$ are supplementary	4) If 2 angles form a linear pair, then they are supplementary
5) $\angle EDB \cong \angle GFC$	5) Supplements of $\cong$ angles are $\cong$
6) $\triangle DBE \cong \triangle FCG$	6) ASA $\cong$ ASA
7) $\angle B \cong \angle C$	7) CPCTC
8) $\overline{AB} \cong \overline{AC}$	8) If 2 angles of a $\triangle$ are $\cong$ the sides opp those angles are $\cong$
9) $\triangle ABC$ is isosceles	9) An isosceles $\triangle$ has 2 $\cong$ sides

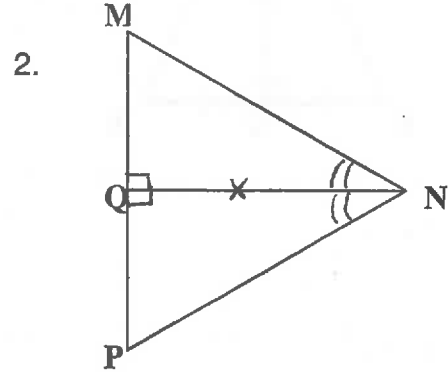
## Corresponding Parts of Congruent Triangles

Remember: When any two triangles are congruent, their corresponding sides are congruent and their corresponding angles are congruent.

- Examples:
- Name 2 triangles that are congruent
  - State the reason why the triangles are congruent
  - Name 3 additional pairs of congruent parts



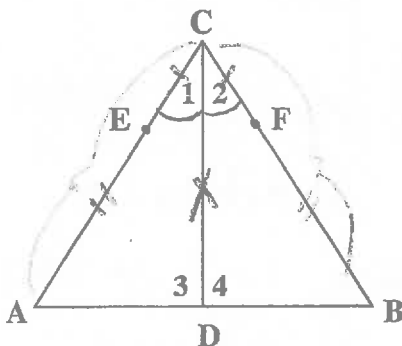
- $\triangle ABC \cong \triangle EDC$
- $SAS \cong SAS$
- $\angle B \cong \angle D, \angle A \cong \angle E$   
 $\overline{AB} \cong \overline{DE}$



- $\triangle QNM \cong \triangle QNP$
- $ASA \cong ASA$
- $\angle M \cong \angle P, \overline{MN} \cong \overline{PN}$   
 $\overline{MQ} \cong \overline{QP}$

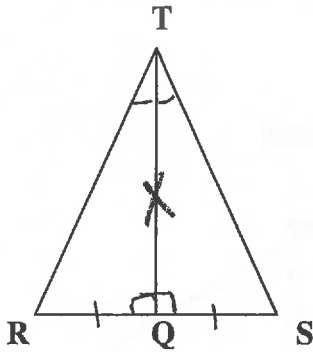
3. Given:  $\angle 1 \cong \angle 2$   
 $\overline{CE} \cong \overline{CF}$   
 $\overline{EA} \cong \overline{FB}$

Prove:  $\angle 3 \cong \angle 4$



Statements	Reasons
1) $\angle 1 \cong \angle 2$	1) given
2) $\overline{CE} \cong \overline{CF}$	2) given
3) $\overline{EA} \cong \overline{FB}$	3) given
4) $\overline{CE} + \overline{EA} = \overline{CF} + \overline{FB}$	4) addition
5) $\overline{CA} = \overline{CE} + \overline{EA}$ $\overline{CB} = \overline{CF} + \overline{FB}$	5) Partition
6) $\overline{CA} \cong \overline{CB}$	6) substitution
7) $\overline{CD} \cong \overline{CD}$	7) Reflexive
8) $\triangle ACD \cong \triangle BCD$	8) $SAS \cong SAS$
9) $\angle 3 \cong \angle 4$	9) CPCTC

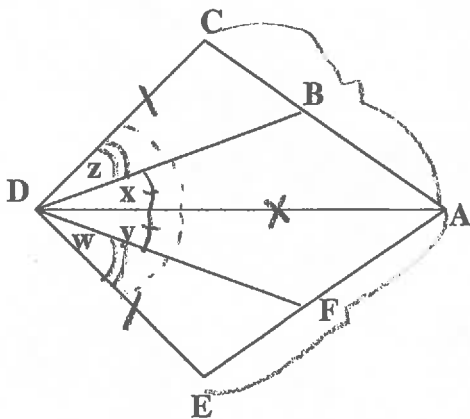
4. If  $\overline{TQ}$  bisects  $\angle RTS$  and  $\overline{TQ} \perp \overline{RS}$ , prove that  $\overline{TQ}$  bisects  $\overline{RS}$ .



Statements	Reasons
1) $\overline{TQ}$ bisects $\angle RTS$	1) given
2) $\overline{TQ} \perp \overline{RS}$	2) given
3) $\angle RTQ \cong \angle STQ$	3) A bisector divides an angle into 2 $\cong$ angles
4) $\angle TQR$ and $\angle TQS$ are right angles	4) $\perp$ lines meet to form right angles
5) $\angle TQR \cong \angle TQS$	5) All right angles are $\cong$
6) $\overline{TQ} \cong \overline{TQ}$	6) Reflexive
7) $\triangle RTQ \cong \triangle STQ$	7) ASA $\cong$ ASA
8) $\overline{RQ} \cong \overline{QS}$	8) CPCTC
9) $\overline{TQ}$ bisects $\overline{RS}$	9) A bisector divides a segment into 2 $\cong$ segments

5. Given:  $\overline{DC} \cong \overline{DE}$   
 $\angle x \cong \angle y$   
 $\angle z \cong \angle w$

Prove:  $\overline{AE} \cong \overline{AC}$



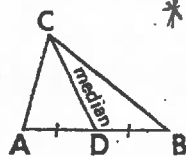
Statements	Reasons
1) $\overline{DC} \cong \overline{DE}$	1) given
2) $\angle x \cong \angle y$	2) given
3) $\angle z \cong \angle w$	3) given
4) $\angle z + \angle x \cong \angle w + \angle y$	4) addition
5) $\angle CDA = \angle z + \angle x$ $\angle EDA = \angle w + \angle y$	5) Partition
6) $\angle CDA \cong \angle EDA$	6) substitution
7) $\overline{AD} \cong \overline{AD}$	7) Reflexive
8) $\triangle CDA \cong \triangle EDA$	8) SAS $\cong$ SAS
9) $\overline{CA} \cong \overline{AE}$	9) CPCTC



# Line Segments Associated With Triangles

**Median of a Triangle:** A line segment that joins any vertex of the triangle to the *midpoint* of the opposite side.

$$\overline{AD} \cong \overline{DB}$$

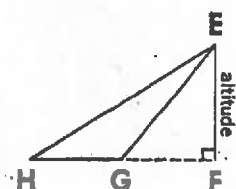
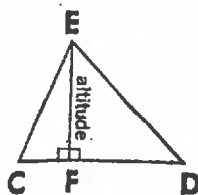


\* D is the midpt of  $\overline{AB}$   
 $\overline{CD}$  is the median drawn from vertex C to side  $\overline{AB}$

\* we can also draw a median from vertex A to side  $\overline{BC}$  ... Every  $\Delta$  has 3 medians

**Altitude of a Triangle:** A line segment drawn from any vertex of the triangle perpendicular to and ending in the line that contains the opposite side.

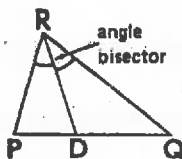
In  $\Delta CED$ , if  $\overline{EF} \perp \overline{CD}$ , then  $\overline{EF}$  is the altitude from vertex E to the opp side



In  $\Delta HEG$ , if  $\overline{EF}$  is  $\perp \overline{HG}$ , the line that contains the side  $\overline{HG}$ , then  $\overline{EF}$  is the altitude from vertex E to opp side

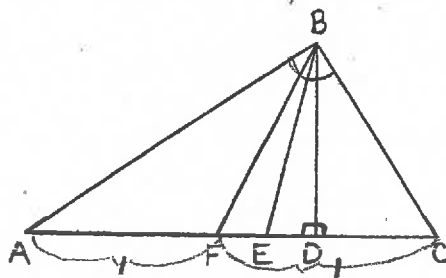
**Angle Bisector of a Triangle:** A line segment that bisects any angle of the triangle and terminates in the side opposite that angle.

In  $\Delta PQR$ , if D is a pt on  $\overline{PQ}$  such that  $\angle PRD \cong \angle QRD$  then  $\overline{RD}$  is the angle bisector from R



\* every  $\Delta$  has 3 angle bis

In a scalene triangle, the *altitude*, the *median*, and the *angle bisector* drawn from any common vertex are three distinct line segments.

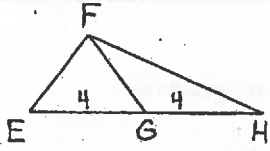


$\overline{BD}$  is the *altitude* from B because  $\overline{BD} \perp \overline{AC}$ ;

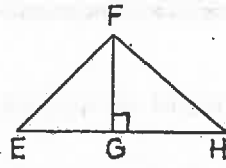
$\overline{BE}$  is the *angle bisector* from B because  $\angle ABE \cong \angle EBC$ ;

$\overline{BF}$  is the *median* from B because F is the midpoint of  $\overline{AC}$ .

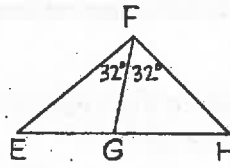
Name the type of line segment that  $\overline{FG}$  is in each triangle.



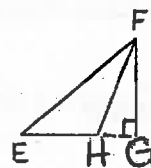
Median  
(middle)



Altitude

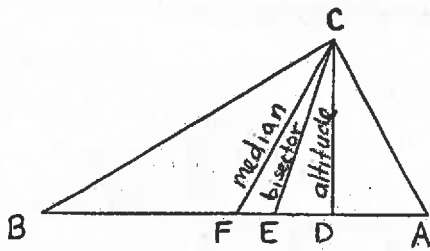


Angle Bisector



Altitude

Polygon ABC is a triangle.  $\overline{CD}$  is an altitude.  $\overline{CE}$  is an angle bisector.  $\overline{CF}$  is a median.



- Name two line segments which are congruent.
- Name two angles which are right angles.
- Name two congruent angles, each of which has its vertex at C.
- Name two line segments which are perpendicular to each other.

$$\underline{\overline{BF} \cong \overline{AF}}$$

$$\underline{\angle CDB \text{ and } \angle CDA}$$

$$\underline{\angle BCE \cong \angle ACE}$$

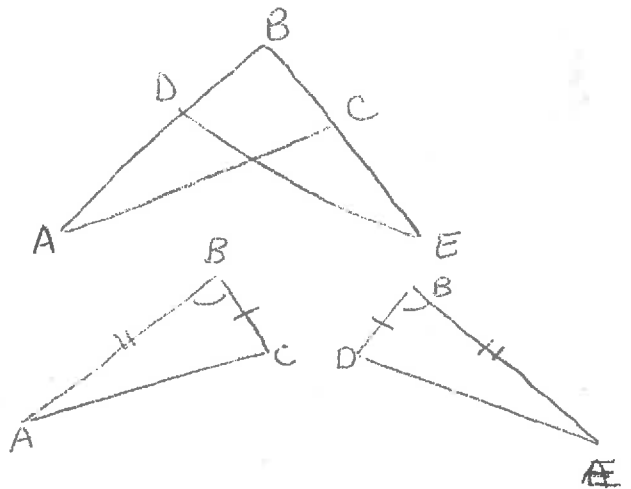
$$\underline{\overline{CD} \perp \overline{BA}}$$

## Overlapping Triangle Proofs

1. Given:  $\overline{BD} \cong \overline{BC}$

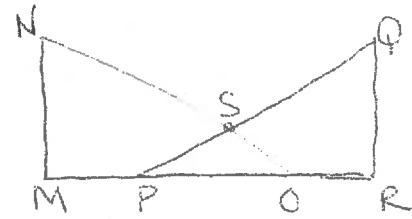
$\overline{DA} \cong \overline{CE}$

Prove:  $\angle A \cong \angle E$

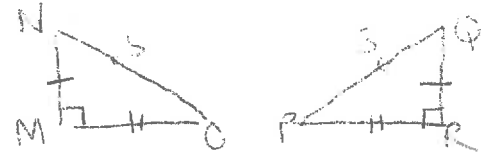


Statements	Reasons
1. $\overline{BD} \cong \overline{BC}$	1. Given
2. $\overline{DA} \cong \overline{CE}$	2. Given
3. $\overline{BA} \cong \overline{BE}$	3. Addition Postulate
4. $\angle B \cong \angle B$	4. Reflexive Postulate
5. $\triangle BAC \cong \triangle BED$	5. SAS $\cong$ SAS
6. $\angle A \cong \angle E$	6. Corresponding parts of $\cong$ $\Delta$ 's are $\cong$ .

\* 2. Given:  $\overline{NM} \cong \overline{QR}$   
 $\overline{NM} \perp \overline{MR}$ ,  $\overline{QR} \perp \overline{MR}$   
 $\overline{MP} \cong \overline{RO}$

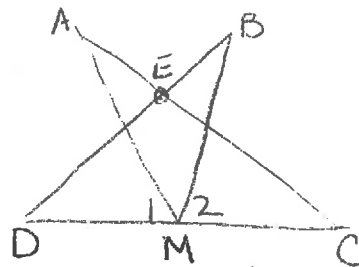


Prove:  $\angle N \cong \angle Q$

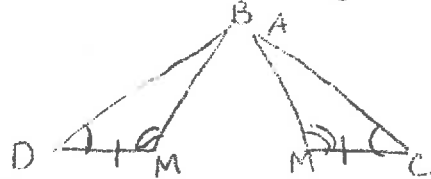


Statements	Reasons
1. $\overline{NM} \cong \overline{QR}$	1. Given
2. $\overline{NM} \perp \overline{MR}$ , $\overline{QR} \perp \overline{MR}$	2. Given
3. $\angle M$ and $\angle R$ are right $\angle$ 's	3. $\perp$ lines form right $\angle$ 's
4. $\angle M \cong \angle R$	4. All right $\angle$ 's are $\cong$ .
5. $\overline{MP} \cong \overline{RO}$	5. Given
6. $\overline{PO} \cong \overline{PO}$	6. Reflexive Postulate
7. $\overline{MO} \cong \overline{RP}$	7. Addition Postulate
8. $\triangle NMO \cong \triangle QRP$	8. SAS $\cong$ SAS
9. $\angle N \cong \angle Q$	9. Corresponding parts of $\cong$ $\Delta$ 's are $\cong$ .

Given :  $\overline{AC}$  &  $\overline{BD}$  intersect at E  
 $\angle D \cong \angle C$   
 M is the midpoint of  $\overline{DC}$   
 $\angle 1 \cong \angle 2$



Prove :  $\overline{DB} \cong \overline{CA}$



Statements	Reasons
1. $\overline{AC}$ & $\overline{BD}$ intersect at E	1. Given
2. $\angle D \cong \angle C$	2. Given
3. M is the midpoint of $\overline{DC}$	3. Given
4. $\overline{DM} \cong \overline{CM}$	4. A midpoint cuts a segment into 2 $\cong$ segments
5. $\angle 1 \cong \angle 2$	5. Given
6. $\angle AMB \cong \angle AMB$	6. Reflexive Postulate
7. $\angle BMD \cong \angle AMC$	7. Addition Postulate
8. $\triangle BMD \cong \triangle AMC$	8. ASA $\cong$ ASA
9. $\overline{DB} \cong \overline{CA}$	9. Corresponding parts of $\cong$ $\Delta$ 's are $\cong$ .

