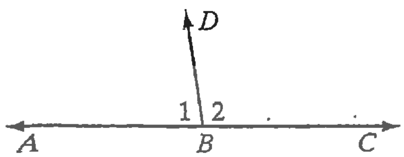
Name ____________________ Class __________ Date __________

Chapter 4-2 Using Postulates and Definitions in Proofs

PART I

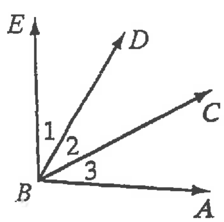
Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. [12]

1. In the given figure, if $m\angle 1 = 5x - 4$ and $m\angle 2 = 3x + 48$, what is $m\angle 1$?



(1) 17 (3) 99
(2) 81 (4) 126

2. In the given figure, $m\angle 1 = m\angle 2 = m\angle 3$. If $m\angle 1 = 7x + 11$ and $m\angle 3 = 12x - 4$, what is $m\angle ABD$?

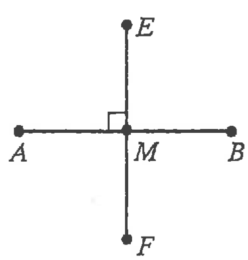


(1) 32 (3) 64
(2) 33 (4) 66

3. Segment $\overline{BD}$ is the perpendicular bisector of $\overline{ABC}$. If $AB = 3x - 30$ and $BC = 2x - 10$, what is the length of $\overline{AC}$?

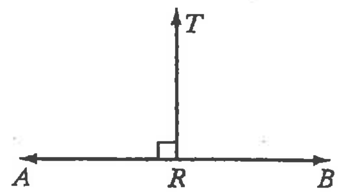
(1) 20 (3) 40
(2) 30 (4) 60

4. In the given figure, $\overline{AB}$ and $\overline{EF}$ are perpendicular bisectors of each other at point M. If $AM = EM$, $AB = 4x + 7$, and $EF = 7x - 20$, what is the length of $\overline{AB}$?



(1) 9 (3) 29
(2) 21.5 (4) 43

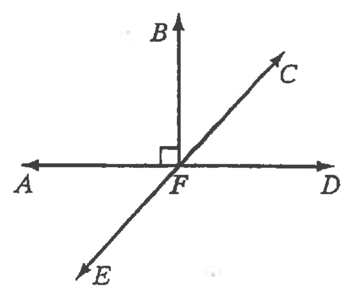
OMIT
5. In the given figure, $\overrightarrow{RT} \perp \overleftrightarrow{AB}$, $m\angle ART = 3x - y$, and $m\angle TRB = 2x + y$.



Which ordered pair represents the values (x, y)?

(1) (36, 18) (3) (45, 45)
(2) (18, 36) (4) (90, 90)

6. In the given figure, $\angle AFB$ is a right angle. If $m\angle AFE = 3y + 3$, and $m\angle CFB = 3y - 3$, what is the value of y?



(1) 15 (3) 42
(2) 30 (4) 48

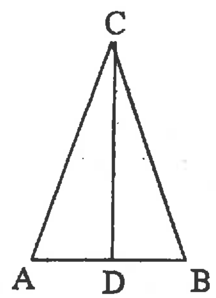
Proof Toolbox

Remember: To prove 2 triangles congruent, you need 3 congruences!!

Here are some ways you can get **2 segments or two angles** congruent:

1. **Midpoint** → Look for word "**midpoint**" in the givens.

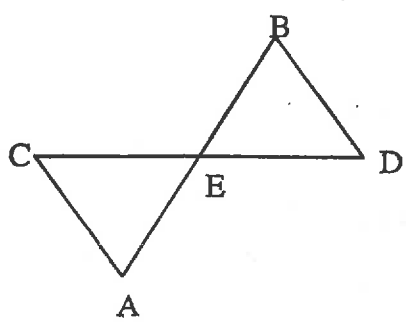
Given: D is the midpoint of $\overline{AB}$



Statements	Reasons
1. D is the midpoint of $\overline{AB}$	1. Given
2. $\overline{AD} \cong \overline{BD}$	2. A midpoint cuts a segment into 2 $\cong$ segments.

2. **Segment Bisector** → Look for word "**bisects**" between 2 segments in givens.
   NOTE: The 2[nd] segment listed is the one that's being bisected.

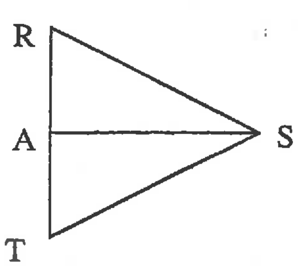
Given:
$\overline{CD}$ bisects $\overline{AB}$



Statements	Reasons
1. $\overline{CD}$ bisects $\overline{AB}$	1. Given
2. $\overline{AE} \cong \overline{BE}$	2. A segment. bisector cuts a segment. into 2 $\cong$ segments.

3. **Angle Bisector** → Look for word "**bisects**" and the **angle** symbol ($\angle$)

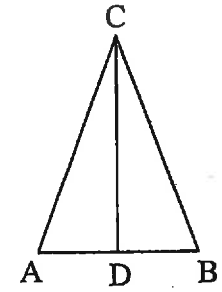
Given: $\overline{AS}$ bisects $\angle RST$



Statements	Reasons
1. $\overline{AS}$ bisects $\angle RST$	1. Given
2. $\angle RSA \cong \angle TSA$	2. An $\angle$ bisector cuts an $\angle$ into 2 $\cong$ $\angle$'s.

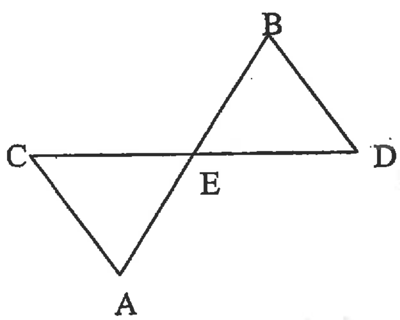
4. **Perpendicular Lines** → Look for perpendicular symbol (⊥) **NOTE:** You must have 2 STEPS!!!

Given:
$\overline{CD} \perp \overline{AB}$



Statements	Reasons
1. $\overline{CD} \perp \overline{AB}$	1. Given
2. $\angle CDA, \angle CDB$ are rt. ∠'s	2. ⊥ lines form right ∠'s
3. $\angle CDA \cong \angle CDB$	3. All right ∠'s are ≅

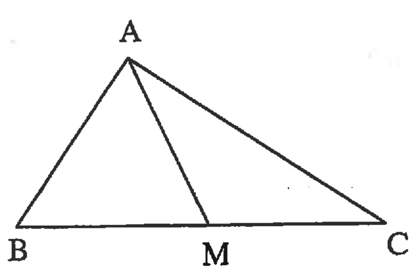
5. **Vertical Angles** → **Will not** be in the givens- you must look for an **"X"** in the diagram. **CAUTION:** Don't use one letter to name vertical angles!!!



Statements	Reasons
1. $\angle CEA \cong \angle DEB$	1. Vertical ∠'s are ≅

6. **Median** → Look for word **"median"** **NOTE:** You must have 2 STEPS!!!

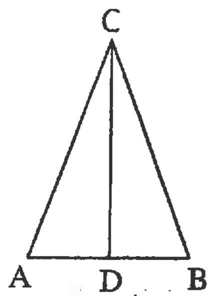
Given: $\overline{AM}$ is median to $\overline{BC}$



Statements	Reasons
1. $\overline{AM}$ is median to $\overline{BC}$	1. Given
2. M is the midpoint of $\overline{BC}$	2. A median of a Δ intersects the opposite side at it's midpoint.
3. $\overline{BM} \cong \overline{CM}$	3. A midpoint cuts a segment into 2 ≅ segments.

7. **Altitude →** Look for word "**altitude**" **NOTE:** You need 3 STEPS!!!

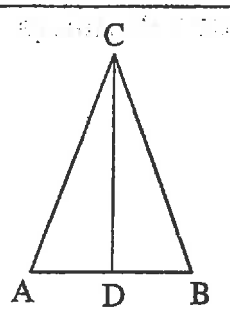
Given: $\overline{CD}$ is the altitude to $\overline{AB}$



Statements	Reasons
1. $\overline{CD}$ is altitude to $\overline{AB}$	1. Given
2. $\overline{CD} \perp \overline{AB}$	2. An altitude of a Δ is drawn ⊥ to the side it intersects.
3. $\angle CDA, \angle CDB$ are rt. ∠'s	3. ⊥ lines form right ∠'s
4. $\angle CDA \cong \angle CDB$	4. All right ∠'s are ≅

8. **Perpendicular Bisector →** 2 things going on: **⊥ lines and seg. bisector**
NOTE: You will need 3 steps!!!

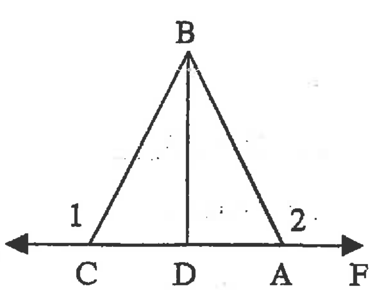
Given: $\overline{CD}$ is the ⊥ bisector to $\overline{AB}$



Statements	Reasons
1. $\overline{CD}$ is ⊥ bisector to $\overline{AB}$	1. Given
2. $\angle CDA, \angle CDB$ are rt. ∠'s	2. ⊥ lines form right ∠'s
3. $\angle CDA \cong \angle CDB$	3. All right ∠'s are ≅
4. $\overline{AD} \cong \overline{BD}$	4. A segment bisector cuts a segment into 2 ≅ segments.

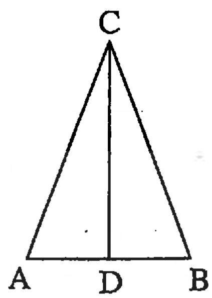
9. **Exterior Angles →** Look for 2 ∠'s **not** inside the 2 Δ's you are working with.
NOTE: You will need 2 STEPS!!!

Given: $\angle 1 \cong \angle 2$

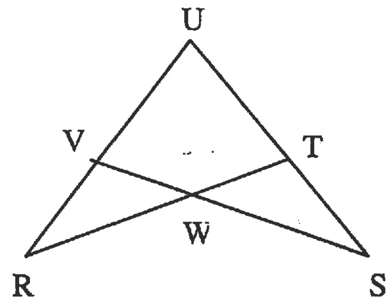


Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. Given
2. $\angle 1$ and $\angle BCD$ are supplementary $\angle 2$ and $\angle BAD$ are supplementary	2. If 2 ∠'s form a linear pair, then they are supplementary.
3. $\angle BCD \cong \angle BAD$	3. If 2 ∠'s are ≅, then their Supplements are ≅.

10. **Sharing a Side or an Angle →** Use **Reflexive** Postulate



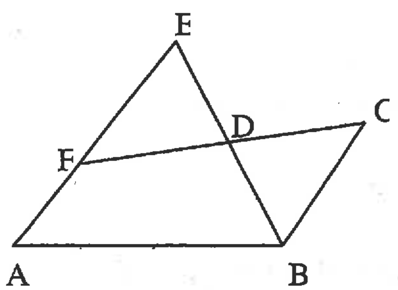
$\overline{CD} \cong \overline{CD}$



$\angle U \cong \angle U$

11. **Substitution →** Look for the **same** segment or angle appearing **twice** in separate statements. **NOTE:** If you cross out the ones that are the same, you will be left with your conclusion.

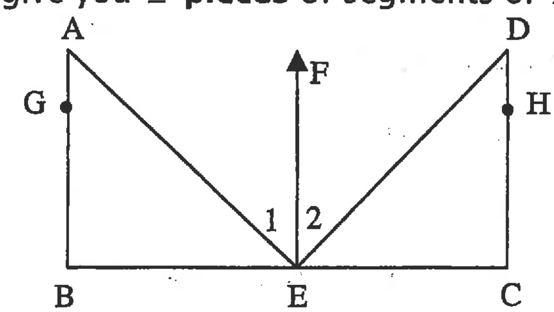
Given: $\overline{AF} \cong \overline{BC}$
$\overline{EF} \cong \overline{AF}$



Statements	Reasons
1. $\overline{AF} \cong \overline{BC}$	1. Given
2. $\overline{EF} \cong \overline{AF}$	2. Given
3. $\overline{EF} \cong \overline{BC}$	3. Substitution Postulate

12. **Subtraction →** They may give you **too much**: A pair of $\angle$'s that are **too big**, or a pair of segments that are **too long**
Addition → They may give you $\cong$ **pieces** of segments or $\angle$'s, but not whole sides or $\angle$'s of Δ's.

Given: $\angle 1 \cong \angle 2$
$\angle BEF \cong \angle CEF$



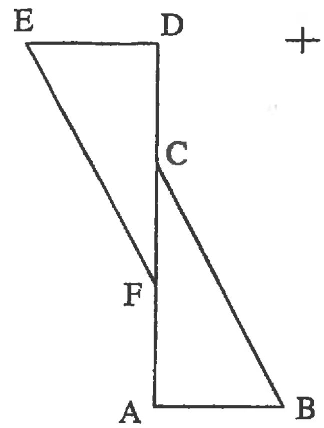
Given: $\overline{BG} \cong \overline{CH}$
$\overline{GA} \cong \overline{HD}$

Statements	Reasons
− 1. $\angle 1 \cong \angle 2$	1. Given
− 2. $\angle BEF \cong \angle CEF$	2. Given
3. $\angle BEA \cong \angle CED$	3. Subtraction Postulate

Statements	Reasons
+ 1. $\overline{BG} \cong \overline{CH}$	1. Given
+ 2. $\overline{GA} \cong \overline{HD}$	2. Given
3. $\overline{BA} \cong \overline{CD}$	3. Addition Postulate

13. **Overlapping Parts Addition and Subtraction** → You may need to use **Reflexive**!

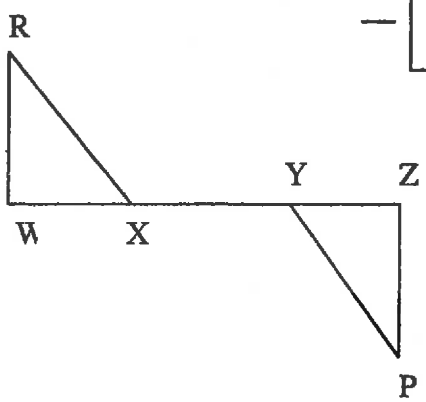
Given: $\overline{DC} \cong \overline{AF}$



Statements	Reasons
+ 1. $\overline{DC} \cong \overline{AF}$	1. Given
2. $\overline{CF} \cong \overline{CF}$	2. Reflexive Postulate
3. $\overline{DF} \cong \overline{AC}$	3. Addition Postulate

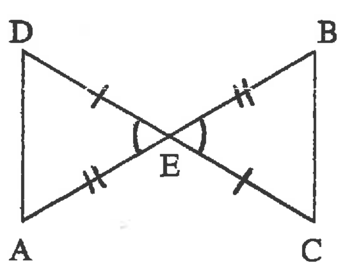
14.

Given: $\overline{WY} \cong \overline{ZX}$



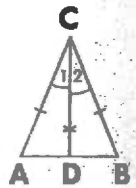
Statements	Reasons
− 1. $\overline{WY} \cong \overline{ZX}$	1. Given
2. $\overline{XY} \cong \overline{XY}$	2. Reflexive Postulate
3. $\overline{WX} \cong \overline{ZY}$	3. Subtraction Postulate

15. If you can prove that 2 Δ's are ≅, but you **need** to prove that **2 segments or 2 ∠'s are ≅**: Use **Corresponding Parts**.



Statements	Reasons
1. $\Delta AED \cong \Delta BEC$	1. SAS ≅ SAS
2. $\overline{AD} \cong \overline{BC}$ (or $\angle A \cong \angle B$, $\angle C \cong \angle D$)	2. Corresponding Parts of ≅ Δ's are ≅.

Note. In steps 1, 3, and 4, we wrote (s. ≅ s.) next to pairs of congruent sides and (a. ≅ a.) next to pairs of congruent angles to help identify the corresponding parts that prove the triangles are congruent.



As an additional aid, you may wish to mark corresponding parts in the diagram with the same number of strokes or arcs. For example, at the right:

$\angle 1$ and $\angle 2$ are marked by single arcs to show $\angle 1 \cong \angle 2$.

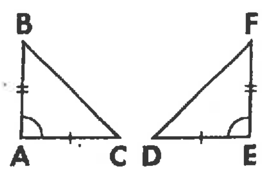
$\overline{AC}$ and $\overline{BC}$ are marked by single strokes to show $\overline{AC} \cong \overline{BC}$.

$\overline{CD}$ is marked with an × to show $\overline{CD} \cong \overline{CD}$.

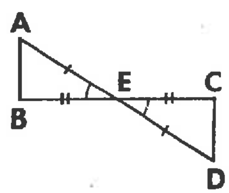
EXERCISES

In 1–6: Pairs of line segments marked with the same number of strokes are congruent. Pairs of angles marked with the same number of arcs are congruent. A line segment or an angle marked with × is congruent to itself by the reflexive property of congruence.

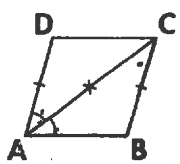
Is the given information sufficient to prove congruent triangles?



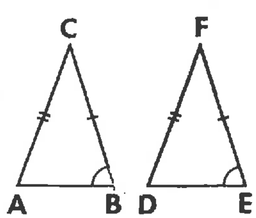
Ex. 1



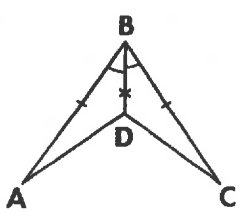
Ex. 2



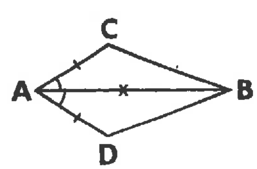
Ex. 3



Ex. 4

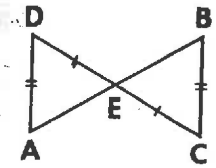


Ex. 5

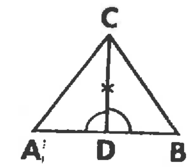


Ex. 6

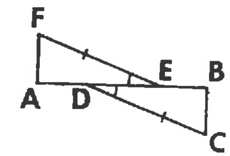
In 7–9, name the pair of corresponding sides or the pair of corresponding angles that would have to be proved congruent (in addition to those pairs marked congruent) in order to prove that the triangles are congruent by s.a.s. ≅ s.a.s.



Ex. 7

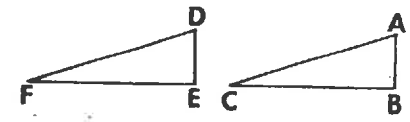


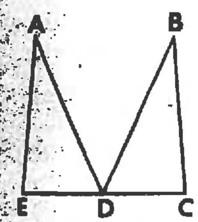
Ex. 8



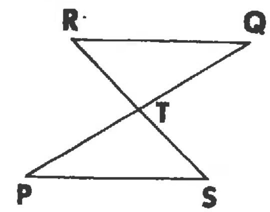
Ex. 9

10. *Given:* $\overline{DE} \cong \overline{AB}$, $\overline{EF} \cong \overline{BC}$, $\angle E$ and $\angle B$ are right angles.
Prove: $\triangle DEF \cong \triangle ABC$.

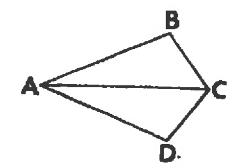




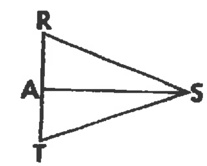
Ex. 11



Ex. 12

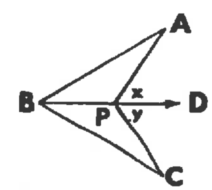


Ex. 13

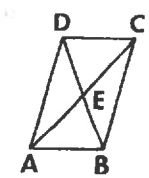


Ex. 14

11. *Given:* $\overline{AE} \cong \overline{BC}$, $\angle E \cong \angle C$, D is the midpoint of $\overline{EC}$.
Prove: $\triangle ADE \cong \triangle BDC$.
12. *Given:* $\overline{RS}$ bisects $\overline{PQ}$ at T, $\overline{PQ}$ bisects $\overline{RS}$ at T.
Prove: $\triangle PTS \cong \triangle QTR$.
13. *Given:* $\overline{AB} \cong \overline{AD}$, $\overline{AC}$ bisects $\angle BAD$.
Prove: $\triangle ABC \cong \triangle ADC$.
14. *Given:* $\overline{AS} \perp \overline{RT}$, A is the midpoint of $\overline{RT}$.
Prove: $\triangle RAS \cong \triangle TAS$.
15. If $\overline{AP} \cong \overline{CP}$, $\angle x \cong \angle y$, and $\overleftrightarrow{BPD}$, prove that $\triangle ABP \cong \triangle CBP$.
16. If $\overline{DB}$ and $\overline{AC}$ bisect each other at E, prove that $\triangle AEB \cong \triangle CED$.



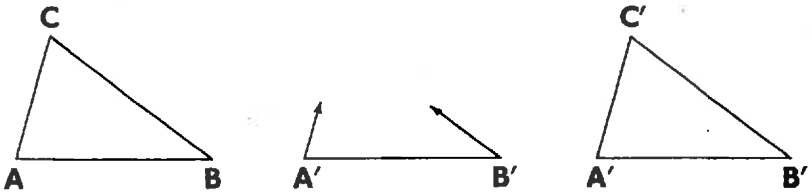
Ex. 15



Ex. 16

4-3 PROVING TRIANGLES CONGRUENT WHEN TWO PAIRS OF ANGLES AND THE INCLUDED SIDE ARE CONGRUENT

In Course I, you saw that we could copy a triangle using a ruler and protractor by drawing only two angles and the side *included* between these angles.



Given *Step 1* *Step 2*

Let us start with $\triangle ABC$. In step 1, copy $\angle A$ to form $\angle A'$, copy the included side $\overline{AB}$ to form the included side $\overline{A'B'}$, and copy $\angle B$ to form $\angle B'$. Although we have copied only two angles and their included side, we see in step 2 that, when we extend the rays at A' and B', these rays must meet at a point that we call C'. The triangle $A'B'C'$ is formed such that $\triangle A'B'C' \cong \triangle ABC$.

This experiment, repeated several times, leads to the following statement whose truth will be assumed without proof:

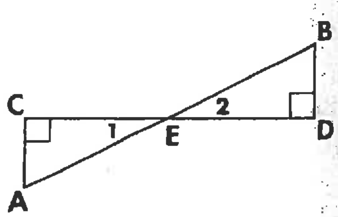
● **POSTULATE 29. Two triangles are congruent if two angles and the included side of one triangle are congruent respectively to two angles and the included side of the other. [a.s.a. ≅ a.s.a.]**

Thus, in $\triangle ABC$ and $\triangle A'B'C'$, if $\angle A \cong \angle A'$, $\overline{AB} \cong \overline{A'B'}$ and $\angle B \cong \angle B'$, it follows that $\triangle ABC \cong \triangle A'B'C'$. We will now use this postulate to prove two triangles congruent.

MODEL PROBLEM

Given: $\overleftrightarrow{CD}$ and $\overleftrightarrow{AB}$ intersect at E.
$\overline{BA}$ bisects $\overline{CD}$. $\overline{AC} \perp \overline{CD}$,
$\overline{BD} \perp \overline{CD}$.

Prove: $\triangle ACE \cong \triangle BDE$.

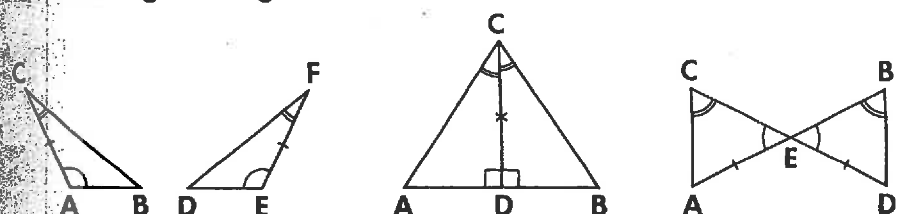


Plan: Prove triangles congruent by showing t a.s.a. ≅ a.s.a. (Insert numbers for every reference to angles.)

Statements	*Reasons*
1. $\overleftrightarrow{CD}$ and $\overleftrightarrow{AB}$ intersect E.	1. Given.
2. $\angle 1$ and $\angle 2$ are vertical angles.	2. Definition of vertical angles.
3. $\angle 1 \cong \angle 2$. (a. ≅ a.)	3. If two angles are vertical angles, they are congruent.
4. $\overline{BA}$ bisects $\overline{CD}$.	4. Given.
5. $\overline{CE} \cong \overline{ED}$. (s. ≅ s.)	5. Definition of bisector of a line segment.
6. $\overline{AC} \perp \overline{CD}$, $\overline{BD} \perp \overline{CD}$.	6. Given.
7. $\angle C$ and $\angle D$ are right angles.	7. Definition of perpendicular lines.
8. $\angle C \cong \angle D$. (a. ≅ a.)	8. If two angles are right angles, they are congruent.
9. $\triangle ACE \cong \triangle BDE$.	9. a.s.a. ≅ a.s.a.

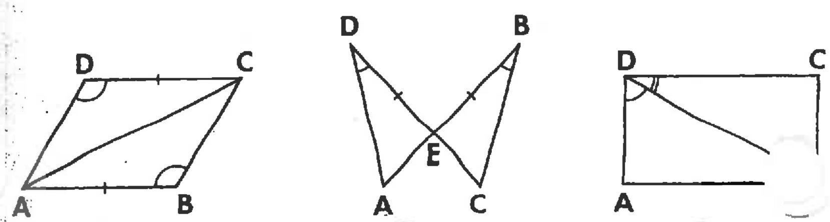
EXERCISES

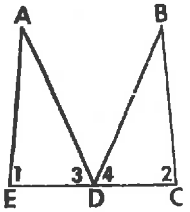
In 1–3, tell whether or not the triangles can be proved congruent by the a.s.a. ≅ a.s.a. postulate, using only the marked congruent parts in establishing the congruence.



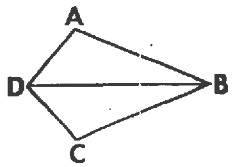
Ex. 1 Ex. 2 Ex. 3

In 4–6, name the pair of corresponding sides or the pair of corresponding angles that would have to be proved congruent (in addition to those pairs marked congruent) in order to prove that the triangles are congruent by a.s.a ≅ a.s.a.

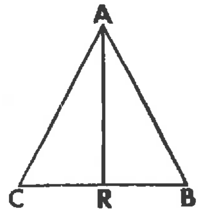




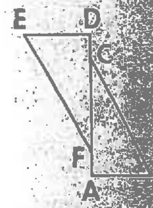
Ex. 7



Ex. 8



Ex. 9

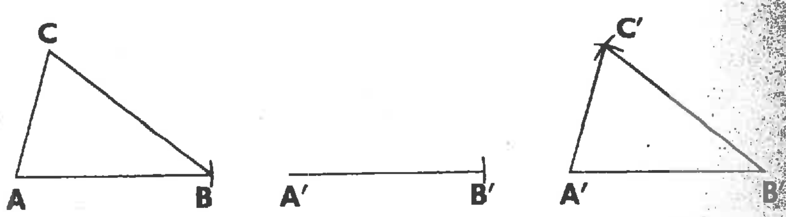


Ex. 10

7. *Given:* $\angle 1 \cong \angle 2$, D is the midpoint of $\overline{EC}$, $\angle 3 \cong \angle 4$.
   Prove: $\triangle AED \cong \triangle BCD$.
8. *Given:* $\overline{DB}$ bisects $\angle ADC$, $\overline{BD}$ bisects $\angle ABC$.
   Prove: $\triangle ADB \cong \triangle CDB$.
9. *Given:* $\overline{AR} \perp \overline{CB}$, $\overline{AR}$ bisects $\angle CAB$.
   Prove: $\triangle ACR \cong \triangle ABR$.
10. *Given:* $\overline{DCFA}$, $\angle E \cong \angle B$, $\overline{ED} \cong \overline{AB}$, $\overline{FD} \perp \overline{DE}$, $\overline{CA} \perp \overline{AB}$.
    Prove: $\triangle DEF \cong \triangle ABC$.

4-4 PROVING TRIANGLES CONGRUENT WHEN THREE PAIRS OF SIDES ARE CONGRUENT

In Course I, you used compasses to construct a triangle by copying the three sides of the triangle. The diagram and explanation that follow show how to do this.



Given $\triangle ABC$. In step 1, copy side $\overline{AB}$ to form side $\overline{A'B'}$. In step 2, using A' as the center of a circle, mark off an arc whose radius is AC. Also, using B' as the center of a different circle, mark off an arc whose radius is BC. The arcs intersect at point C', forming $\triangle A'B'C'$. [You can measure the angles to see that $\angle A \cong \angle A'$, $\angle B \cong \angle B'$, and $\angle C \cong \angle C'$.] Thus, using only three sides, we see that $\triangle ABC \cong \triangle A'B'C'$. Repeated experiments lead to the next statement whose truth is assumed without

POSTULATE 30. Two triangles are congruent if the three sides of one triangle are congruent respectively to the three sides of the other. [s.s.s. ≅ s.s.s.]

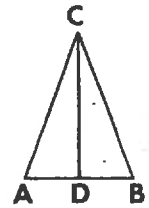
Thus, in $\triangle ABC$ and $\triangle A'B'C'$, if $\overline{AB} \cong \overline{A'B'}$, $\overline{BC} \cong \overline{B'C'}$, and $\overline{CA} \cong \overline{C'A'}$, it follows that $\triangle ABC \cong \triangle A'B'C'$. We will now use this postulate to prove two triangles congruent.

MODEL PROBLEM

Given: Isosceles triangle ABC with $\overline{CA} \cong \overline{CB}$.
D is the midpoint of base $\overline{AB}$.

Prove: $\triangle ACD \cong \triangle BCD$.

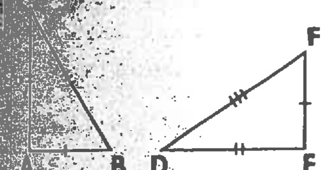
Plan: Prove the triangles congruent by showing that s.s.s. ≅ s.s.s.

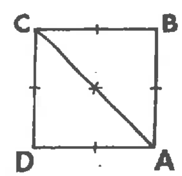


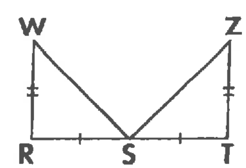
Statements	*Reasons*
1. Isosceles triangle ABC, $\overline{CA} \cong \overline{CB}$. (s. ≅ s.)	1. Given.
2. D is the midpoint of $\overline{AB}$.	2. Given.
3. $\overline{AD} \cong \overline{DB}$. (s. ≅ s.)	3. Definition of midpoint.
4. $\overline{CD} \cong \overline{CD}$. (s. ≅ s.)	4. Reflexive property of congruence.
5. $\triangle ACD \cong \triangle BCD$.	5. s.s.s. ≅ s.s.s.

EXERCISES

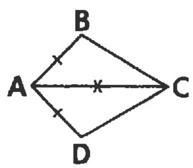
In 1–3, tell whether or not the triangles can be proved congruent using only the marked congruent parts in establishing the congruence. Give the reason for your answer.



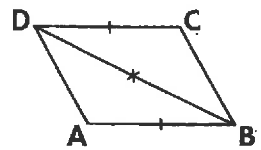




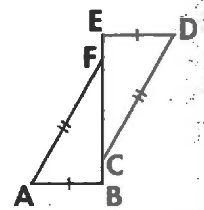
In 4–6, name the pair of corresponding sides that would have to be proved congruent (in addition to those pairs marked congruent) in order to prove that the triangles are congruent by s.s.s. ≅ s.s.s.



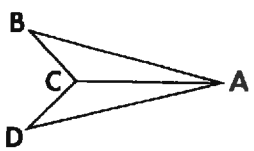
Ex. 4



Ex. 5



Ex. 6



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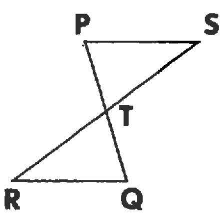
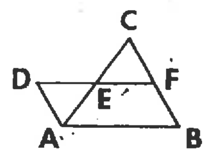


Ex. 8



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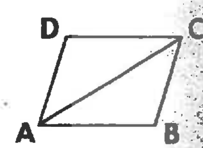


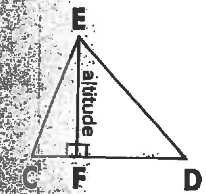
Ex. 10

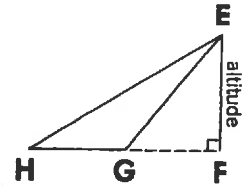
7. *Given:* $\overline{AB} \cong \overline{AD}$, $\overline{CB} \cong \overline{CD}$.
   Prove: $\triangle ABC \cong \triangle ADC$.
8. *Given:* T is the midpoint of $\overline{PQ}$, $\overline{PQ}$ bisects $\overline{RS}$, $\overline{RQ} \cong \overline{SP}$.
   Prove: $\triangle RTQ \cong \triangle STP$.
9. *Given:* $\overline{AC}$ and $\overline{DF}$ bisect each other at E, $\overline{AD} \cong \overline{CF}$.
   Prove: $\triangle DEA \cong \triangle FEC$.
10. If both pairs of opposite sides of quadrilateral $ABCD$ are congruent, prove that $\triangle ABC \cong \triangle CDA$.

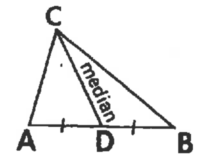
4-5 MORE LINE SEGMENTS ASSOCIATED WITH TRIANGLES

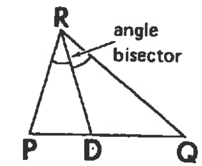
In Chapter 1, an ***altitude of a triangle*** was defined as a line segment drawn from any vertex of the triangle, perpendicular to and ending in the line that contains the opposite side. Thus, every triangle has three altitudes. In $\triangle CED$ and $\triangle HEF$ below, the altitude $\overline{EF}$ is shown for both triangles.

There are [illegible] line segments that are useful in the study of triangles, namely *med*[illegible] [illegible]nd *angle bisector* of a triangle.









Definition. A ***median of a triangle*** is a line segment that joins any vertex of the triangle to the midpoint of the opposite side.

In $\triangle ABC$ above, if D is the midpoint of $\overline{AB}$, then $\overline{CD}$ is the median drawn from vertex C to side $\overline{AB}$. We may also draw a median from vertex A to the midpoint of side $\overline{BC}$, and a median from vertex B to the midpoint of side $\overline{AC}$. Thus, every triangle has three medians.

> To construct a median of a given triangle.
> *See Chapter 16, Construction 8.*

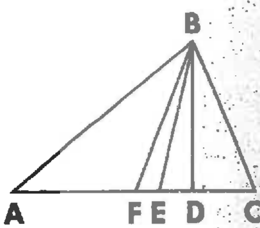
Definition. An ***angle bisector of a triangle*** is a line segment that bisects any angle of the triangle and terminates in the side opposite that angle.

In $\triangle PQR$ above, if $\angle PRD \cong \angle QRD$ and D is a point on side $\overline{PQ}$, then $\overline{RD}$ is the bisector of $\angle PRQ$ in $\triangle PQR$. An angle bisector may also be drawn from the vertex of $\angle P$ to some point on $\overline{RQ}$; and an angle bisector may also be drawn from the vertex of $\angle Q$ to some point on $\overline{PR}$. Thus, every triangle has three angle bisectors.

> To construct the bisector of an angle of a given triangle.
> *See Chapter 16, Construction 9.*

In a scalene triangle, the *altitude*, the *median*, and the *angle* [illegible]*ctor* drawn from any common vertex are three distin[illegible] [illegible]ne[illegible] ments.

In $\triangle ABC$, from the common vertex B, three line segments are drawn:

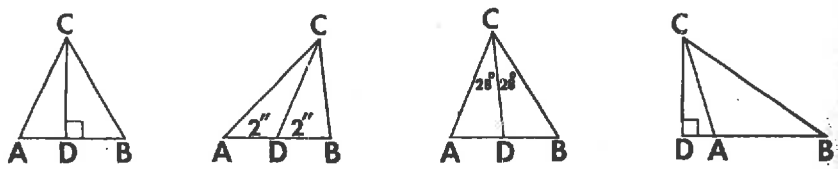


$\overline{BD}$ is the *altitude* from B because $\overline{BD} \perp \overline{AC}$;
$\overline{BE}$ is the *angle bisector* from B because $\angle ABE \cong \angle EBC$;
$\overline{BF}$ is the *median* from B because F is the midpoint of $\overline{AC}$.

In some special triangles (such as the isosceles triangle and the equilateral triangle), some of these segments *coincide*, that is, fall on the same line. You will see these examples later.

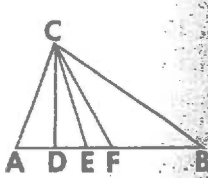
EXERCISES

1. In each triangle below, name the type of line segment that $\overline{CD}$ is.



Ex. 1

2. Polygon ABC is a triangle. $\overline{CD}$ is an altitude. $\overline{CE}$ is an angle bisector. $\overline{CF}$ is a median.



   a. Name two congruent angles, each of which has its vertex at C.
   b. Name two line segments that are congruent.
   c. Name two line segments that are perpendicular to each other.
   d. Name two angles that are right angles.

4-6 MORE PRACTICE IN PROVING TRIANGLES CONGRUENT

Methods of Proving Triangles Congruent

To prove that two triangles are congruent, prove that any one of the following statements is true:

1. Two sides and the included angle of one triangle are congruent respectively to two sides and the included angle of the other. [s.a.s. ≅ s.a.s.]

2. Two angles and the included side of one triangle are congruent respectively to two angles and the included side of the other. [a.s.a. ≅ a.s.a.]
3. Three sides of one triangle are congruent respectively to the three sides of the other. [s.s.s. ≅ s.s.s.]

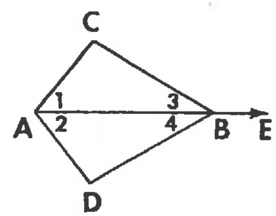
Analyzing a Congruence Problem

For the three given congruence postulates, a process of *analysis* can help to determine which postulate can be used to prove that two triangles are congruent. Let us see how to perform such an analysis for the following congruence problems.

MODEL PROBLEMS

1. *Given:* $\overleftrightarrow{ABE}$ bisects $\angle CAD$.
   $\angle CBE \cong \angle DBE$.

   Prove: $\triangle ACB \cong \triangle ADB$.



Since $\overleftrightarrow{ABE}$ bisects $\angle CAD$, $\angle 1 \cong \angle 2$, giving us one pair of congruent angles. Also, $\overline{AB}$ is a common side in both triangles. Thus, $\overline{AB} \cong \overline{AB}$ by the reflexive property of congruence, giving us a pair of congruent sides. We will attempt to use either the a.s.a. postulate or the s.a.s. postulate.

To use the s.a.s. postulate, we must prove that $\overline{AC} \cong \overline{AD}$. Having no information about these sides, we cannot use the s.a.s. postulate.

To use the a.s.a. postulate, we must prove that $\angle 3 \cong \angle 4$.

Since we know from the *given* that $\angle CBE \cong \angle DBE$ and that we have a straight line, $\overleftrightarrow{ABE}$, we can show that $\angle 3$ must be congruent to $\angle 4$ because two angles that are supplementary to congruent angles are congruent.

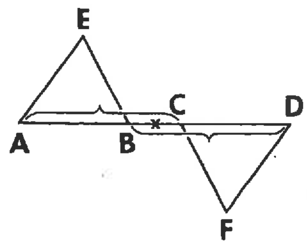
Therefore, we prove $\triangle ACB \cong \triangle ADB$ by the a.s.a. postulate as follows:

Statements	*Reasons*
1. $\overleftrightarrow{ABE}$ bisects $\angle CAD$.	1. Given.
2. $\angle 1 \cong \angle 2$. (a. ≅ a.)	2. Definition of angle bisector.
3. $\overline{AB} \cong \overline{AB}$. (s. ≅ s.)	3. Reflexive property of congruence.
4. $\overleftrightarrow{ABE}$.	4. Given.
5. $\angle CBE \cong \angle DBE$.	5. Given.

6. $\angle 3$ is supplementary to $\angle CBE$. $\angle 4$ is supplementary to $\angle DBE$.	6. If two angles form a linear pair, the angles are supplementary.
7. $\angle 3 \cong \angle 4$. (a. ≅ a.)	7. If two angles are supplements of congruent angles, they are congruent.
8. $\triangle ACB \cong \triangle ADB$.	8. a.s.a. ≅ a.s.a.

2. *Given:* $\overline{ABCD}$.
$\overline{AE} \cong \overline{DF}$.
$\angle A \cong \angle D$.
$\overline{AC} \cong \overline{DB}$.

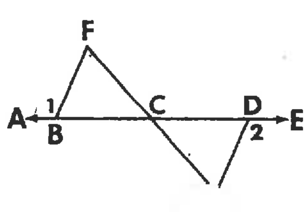
Prove: $\triangle AEB \cong \triangle DFC$.



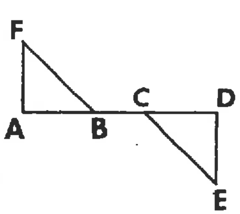
Plan: Prove the triangles congruent by showing that s.a.s. ≅ s.a.s.
To do this, it is necessary to prove that $\overline{AB} \cong \overline{DC}$.

Statements	*Reasons*
1. $\overline{AE} \cong \overline{DF}$. (s. ≅ s.)	1. Given.
2. $\angle A \cong \angle D$. (a. ≅ a.)	2. Given.
3. $\overline{ABCD}$.	3. Given.
4. $\overline{AC} \cong \overline{DB}$.	4. Given.
5. $\overline{BC} \cong \overline{BC}$.	5. Reflexive property of congruence.
6. $\overline{AC} - \overline{BC} \cong \overline{DB} - \overline{BC}$, or $\overline{AB} \cong \overline{DC}$. (s. ≅ s.)	6. Subtraction postulate of congruent segments.
7. $\triangle AEB \cong \triangle DFC$.	7. s.a.s. ≅ s.a.s.

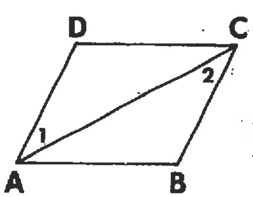
EXERCISES



Ex.

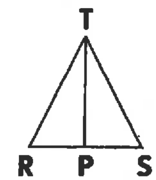


Ex. 2

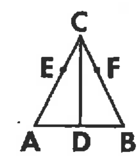


Ex. 3

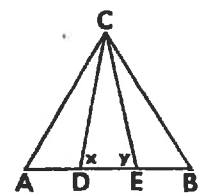
1. *Given:* $\overleftrightarrow{ABCDE}$ and $\overline{FCG}$, C is the midpoint of $\overline{BD}$, $\angle 1 \cong \angle 2$.
   Prove: $\triangle BFC \cong \triangle DGC$.
2. *Given:* $\overline{ABCD}$, $\overline{FA} \perp \overline{AD}$, $\overline{ED} \perp \overline{AD}$, $\overline{AF} \cong \overline{DE}$, $\overline{AC} \cong \overline{DB}$.
   Prove: $\triangle ABF \cong \triangle DCE$.
3. *Given:* In quadrilateral $ABCD$, $\overline{AD} \cong \overline{CB}$ and $\angle 1 \cong \angle 2$.
   Prove: $\triangle ADC \cong \triangle CBA$.



Ex. 4

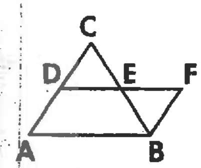


Ex. 5

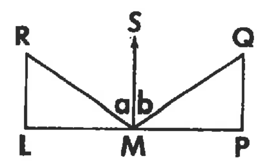


Ex. 6

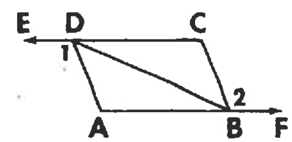
4. *Given:* Isosceles $\triangle RST$ with $\overline{RT} \cong \overline{ST}$, $\overline{TP}$ is a median to base $\overline{RS}$.
   Prove: $\triangle RTP \cong \triangle STP$.
5. *Given:* In triangle ABC, $\overline{CD}$ is a median to $\overline{AB}$, $\overline{CE} \cong \overline{CF}$, $\overline{EA} \cong \overline{FB}$.
   Prove: $\triangle ACD \cong \triangle BCD$.
6. *Given:* Points D and E divide $\overline{AB}$ into three congruent parts, $\overline{CD} \cong \overline{CE}$, $\angle x \cong \angle y$.
   Prove: $\triangle ACD \cong \triangle BCE$.



Ex. 7



Ex. 8



Ex. 9

7. *Given:* E is the midpoint of $\overline{BC}$, $\angle ACB \cong \angle FBC$, $\overline{AD} \cong \overline{CD}$, $\overline{FB} \cong \overline{AD}$.
   Prove: $\triangle CDE \cong \triangle BFE$.
8. *Given:* $\overrightarrow{MS}$ is the perpendicular bisector of $\overline{LP}$, $\overline{RM} \cong \overline{QM}$, $\angle a \cong \angle b$.
   Prove: $\triangle RLM \cong \triangle QPM$.
9. *Given:* $\overrightarrow{CDE}$ and $\overrightarrow{ABF}$, $\angle 1 \cong \angle 2$, $\angle EDB \cong \angle FBD$.
   Prove: $\triangle ADB \cong \triangle CBD$.

Prove: Two right triangles are congruent if the legs of one t gle are congruent to the legs of the other triangle.

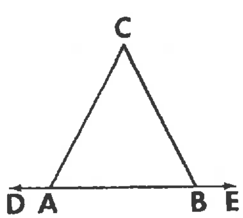
Statements	Reasons
1. $\overline{CA} \cong \overline{CB}$.	1. Given.
2. $\angle A \cong \angle B$. (a. ≅ a.)	2. If two sides of a triangle are congruent, the angles opposite these sides are congruent.
3. $\overline{AD} \cong \overline{BE}$. (s. ≅ s.)	3. Given.
4. M is the midpoint of $\overline{AB}$.	4. Given.
5. $\overline{AM} \cong \overline{BM}$. (s. ≅ s.)	5. A midpoint divides a line segment into two congruent parts.
6. $\triangle ADM \cong \triangle BEM$.	6. s.a.s. ≅ s.a.s.
7. $\overline{MD} \cong \overline{ME}$.	7. Corresponding parts of congruent triangles are congruent.

EXERCISES

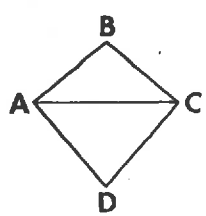
Numerical and Algebraic Applications

1. In $\triangle ABC$, if $\overline{CA} \cong \overline{CB}$ and $m\angle A = 50$, find $m\angle B$.
2. In triangle ABC, $\overline{AB} \cong \overline{BC}$. If $AB = 5x$ and $BC = 2x + 18$, find AB and BC.
3. In isosceles $\triangle ABC$, $\overline{AB} \cong \overline{BC}$. If $AB = 5x + 10$, $BC = 3x + 40$, and $AC = 2x + 30$, find the length of each side of the triangle.
4. In triangle ABC, $\overline{AB} \cong \overline{BC}$. If $m\angle A = 7x$ and $m\angle C = 2x + 50$, find $m\angle A$ and $m\angle C$.
5. In triangle EFG, $\overline{EF} \cong \overline{FG}$. If $m\angle E = 4x + 50$, $m\angle F = 2x + 60$, and $m\angle G = 14x + 30$, find $m\angle E$, $m\angle F$, and $m\angle G$.

Proofs



Ex. 6



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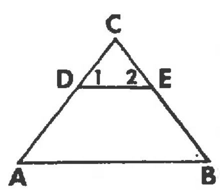
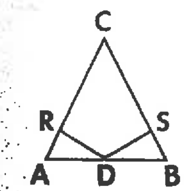
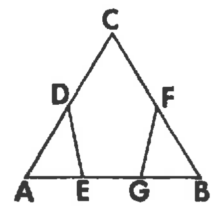


Ex. 8

6. *Given:* $\triangle ABC$ with $\overline{CA} \cong \overline{CB}$ and $\overleftrightarrow{DABE}$.
   Prove: $\angle CAD \cong \angle CBE$.
7. *Given:* Isosceles triangles ABC and ADC have the common base $\overline{AC}$.
   Prove: $\angle BAD \cong \angle BCD$.
8. If $\overline{CA} \cong \overline{CB}$, and $\overline{DA} \cong \overline{EB}$, prove that $\angle 1 \cong \angle 2$.



Ex. 9



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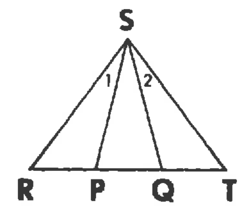
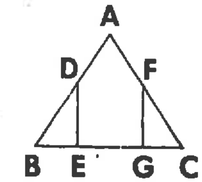
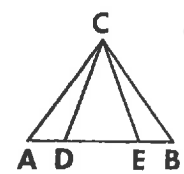


Ex. 11

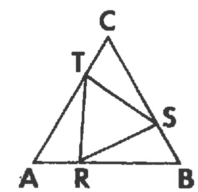


Ex. 12

9. *Given:* In $\triangle ABC$, $\overline{CA} \cong \overline{CB}$, $\overline{AR} \cong \overline{BS}$, $\overline{DR} \perp \overline{AC}$, and $\overline{DS} \perp \overline{BC}$.
   Prove: $\overline{DR} \cong \overline{DS}$.
10. In isosceles triangle ABC, D and F are midpoints of the congruent legs. E and G are the trisection points of the base ($\overline{AE} \cong \overline{EG} \cong \overline{GB}$). Prove that $\overline{DE} \cong \overline{FG}$.
11. Given $\overline{RPQT}$, $\overline{SR} \cong \overline{ST}$, and $\angle 1 \cong \angle 2$, prove that $\triangle PSQ$ is an isosceles triangle.
12. In $\triangle ABC$, $\overline{AB} \cong \overline{AC}$, $\overline{DE} \perp \overline{BC}$, $\overline{FG} \perp \overline{BC}$, and $\overline{BG} \cong \overline{CE}$. Prove that $\overline{BD} \cong \overline{CF}$.



Ex. 13



Ex. 14

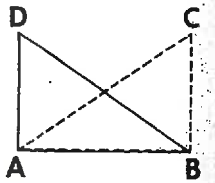
13. Given $\overline{AD} \cong \overline{BE}$, $\overline{CD} \cong \overline{CE}$, and $\overline{ADEB}$, prove that $\overline{AC} \cong \overline{BC}$.
14. If $\triangle ABC$ is an equilateral triangle and $\overline{CT} \cong \overline{AR} \cong \overline{BS}$, prove:
    a. $\overline{TA} \cong \overline{RB} \cong \overline{SC}$
    b. $\triangle TAR \cong \triangle RBS \cong \triangle SCT$
    c. $\overline{TR} \cong \overline{RS} \cong \overline{ST}$
    d. $\triangle TRS$ is an equilateral triangle.
15. *Prove:* The line segments joining the midpoint of the base of an isosceles triangle to the midpoints of the legs are congruent.

4-9 USING TWO PAIRS OF CONGRUENT TRIANGLES

Sometimes, it is impossible to use the *given* in order to prove immediately that a particular pair of triangles is congruent. In such cases, the *given* may contain enough information to first prove another pair of triangles congruent. Then, corresponding congruent parts in these congruent triangles may be used to prove the original pair of triangles congruent. See how this is done in the following example.

4-10 PROVING OVERLAPPING TRIANGLES CONGRUENT

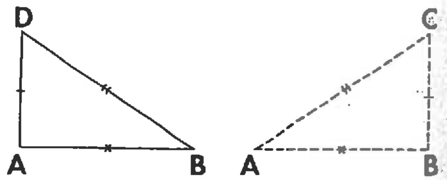
If we are given that $\overline{AD} \cong \overline{BC}$ and $\overline{DB} \cong \overline{CA}$, can we prove that $\triangle DAB \cong \triangle CBA$?



Since these two triangles overlap, you may find it easier to visualize them by using one of the following devices:

1: Outline one of the triangles with a solid line, and the other with a dotted line, as shown above, *or*

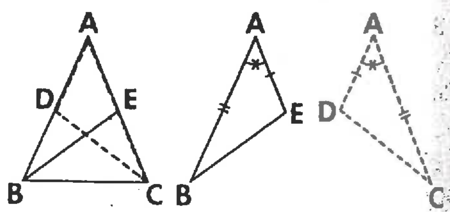
2. Separate the triangles, as shown at the right.



MODEL PROBLEMS

1. *Given:* In $\triangle ABC$, $\overline{AB} \cong \overline{AC}$. $\overline{CD}$ and $\overline{BE}$ are medians.
   Prove: $\overline{BE} \cong \overline{CD}$.
   Plan: Prove $\triangle ABE \cong \triangle ACD$ by s.a.s. ≅ s.a.s. Then, $\overline{BE}$ and $\overline{CD}$ are corresponding parts of these congruent triangles.

Separate the Triangles



Statements	*Reasons*
1. $\overline{AB} \cong \overline{AC}$. (s. ≅ s.)	1. Given.
2. $\angle A \cong \angle A$. (a. ≅ a.)	2. Reflexive property of congruence.
3. $\overline{CD}$ and $\overline{BE}$ are medians.	3. Given.
4. D is the midpoint of $\overline{AB}$. E is the midpoint of $\overline{AC}$.	4. Definition of median of a triangle.
5. $AD = \frac{1}{2}AB$. $AE = \frac{1}{2}AC$.	5. Definition of midpoint.
6. $AD = AE$.	6. Halves of equal quantities are equal.
7. $\overline{AD} \cong \overline{AE}$. (s. ≅ s.)	7. Definition of congruent segments.
8. $\triangle ABE \cong \triangle ACD$.	8. s.a.s. ≅ s.a.s.
9. $\overline{BE} \cong \overline{CD}$.	9. Corresponding parts of congruent triangles are congruent.

2. Using the results of Model Problem 1, find the length of $\overline{BE}$ if $BE = 5x - 8$ and $CD = 3x + 12$.

Solution

(1) Since we proved the segments were congruent, they are equal in length: $BE = CD$

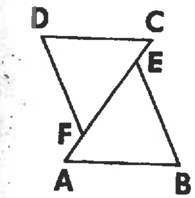
(2) Substitute the given expressions: $5x - 8 = 3x + 12$

(3) Solve for x: $2x = 20$, $x = 10$

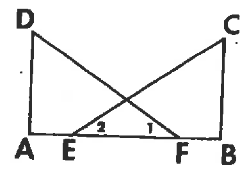
(4) Thus, $BE = 5x - 8 = 5(10) - 8 = 42$.

Answer: $BE = 42$

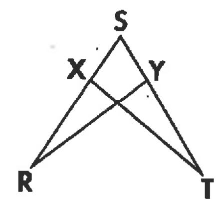
EXERCISES



Ex. 1

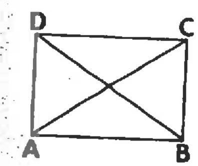


Ex. 2

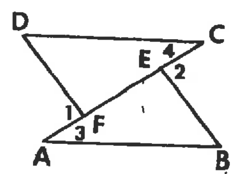


Ex. 3

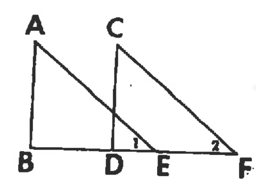
1. *Given:* $\overline{AFEC}$, $\overline{DC} \cong \overline{BA}$, $\overline{DF} \cong \overline{BE}$, and $\overline{CE} \cong \overline{AF}$.
   Prove: $\triangle AEB \cong \triangle CFD$.
2. *Given:* $\overline{AEFB}$, $\overline{CE} \cong \overline{DF}$, $\angle 1 \cong \angle 2$, $\overline{AE} \cong \overline{BF}$.
   Prove: $\triangle AFD \cong \triangle BEC$.
3. *Given:* $\overline{SXR}$, $\overline{SYT}$, $\overline{SX} \cong \overline{SY}$, $\overline{XR} \cong \overline{YT}$.
   Prove: $\triangle RSY \cong \triangle TSX$.



Ex. 4

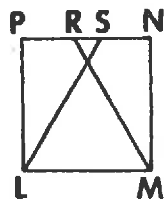


Ex. 5

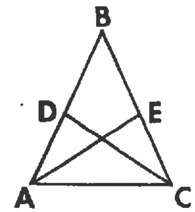


Ex. 6

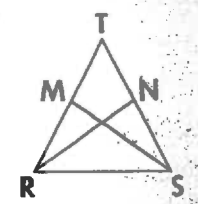
4. *Given:* $\overline{DA} \cong \overline{CB}$, $\overline{DA} \perp \overline{AB}$, $\overline{CB} \perp \overline{AB}$.
   Prove: $\triangle DAB \cong \triangle CBA$.
5. *Given:* $\overline{AFEC}$, $\overline{AF} \cong \overline{EC}$, $\angle 3 \cong \angle 4$, $\angle 1 \cong \angle 2$.
   Prove: $\triangle ABE \cong \triangle CDF$.
6. *Given:* $\overline{AB} \perp \overline{BF}$, $\overline{CD} \perp \overline{BF}$, $\overline{BD} \cong \overline{FE}$, $\angle 1 \cong \angle 2$.
   Prove: $\triangle ABE \cong \triangle CDF$.



Ex. 7

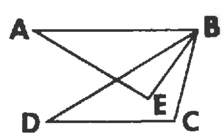


Ex. 8

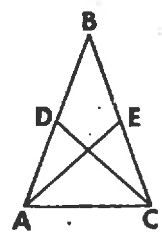


Ex. 9

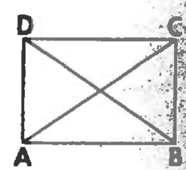
7. *Given:* $\overline{LP} \perp \overline{PN}$, $\overline{MN} \perp \overline{PN}$, $\overline{LP} \cong \overline{MN}$, $\overline{PR} \cong \overline{NS}$, and $\overline{PRSN}$.
   Prove: $\triangle LPS \cong \triangle MNR$.
8. *Given:* $\angle BAC \cong \angle BCA$, $\overline{CD}$ bisects $\angle BCA$, $\overline{AE}$ bisects $\angle BAC$.
   Prove: $\triangle ADC \cong \triangle CEA$.
9. *Given:* $\overline{TR} \cong \overline{TS}$, $\overline{MR} \cong \overline{NS}$.
   Prove: $\triangle RTN \cong \triangle STM$.



Ex. 10

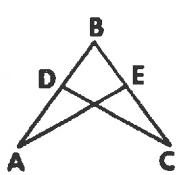


Ex. 11

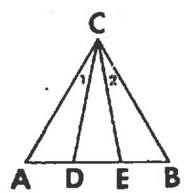


Ex. 12

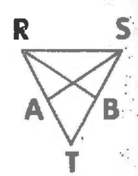
10. *Given:* $\overline{AB} \cong \overline{DB}$, $\angle A \cong \angle D$, $\angle DBA \cong \angle CBE$.
    Prove: $\triangle ABE \cong \triangle DBC$.
11. *Given:* $\overline{DA} \cong \overline{EC}$ and $\overline{DC} \cong \overline{EA}$.
    Prove: **a.** $\triangle CAD \cong \triangle ACE$. **b.** $\angle DCA \cong \angle EAC$.
12. *Given:* $\overline{DA} \perp \overline{AB}$, $\overline{CB} \perp \overline{AB}$, and $\overline{AD} \cong \overline{BC}$.
    Prove: **a.** $\triangle DAB \cong \triangle CBA$. **b.** $\overline{AC} \cong \overline{BD}$.



Ex. 13

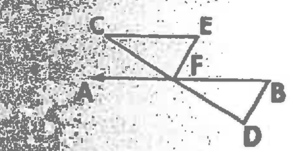


Ex. 14

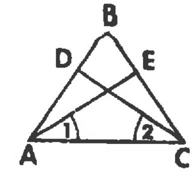


Ex. 15

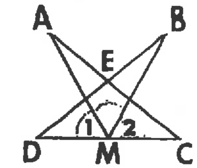
13. *Given:* $\overline{ADB}$, $\overline{BEC}$, $\overline{BD} \cong \overline{BE}$, and $\overline{DA} \cong \overline{EC}$.
    Prove: **a.** $\triangle DBC \cong \triangle EBA$. **b.** $\angle A \cong \angle C$.
14. *Given:* $\overline{ADEB}$, $\overline{AC} \cong \overline{BC}$, $\overline{CE} \cong \overline{CD}$, and $\overline{AE} \cong \overline{BD}$.
    Prove: **a.** $\triangle ACE \cong \triangle BCD$. **b.** $\angle 1 \cong \angle 2$.
15. If $\overline{RT} \cong \overline{ST}$ and median $\overline{RB} \cong$ median $\overline{SA}$, prove that



Ex. 16



Ex. 17

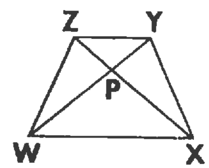


Ex. 18

16. Given $\overrightarrow{BFA}$, $\overline{CFD}$, $\angle ECF \cong \angle CFA$, $\overline{CF} \cong \overline{FD}$, and $\overline{CE} \cong \overline{FB}$, prove that $\overline{EF} \cong \overline{BD}$.
17. Given $\overline{ADB}$, $\overline{BEC}$, $\overline{BD} \cong \overline{BE}$, and $\overline{DA} \cong \overline{EC}$, prove that $\angle 1 \cong \angle 2$.
18. $\overline{AC}$ and $\overline{BD}$ intersect at E, $\angle D \cong \angle C$, M is the midpoint of $\overline{DC}$, and $\angle 1 \cong \angle 2$, prove that $\overline{DB} \cong \overline{CA}$.
19. *Prove:* The medians to the legs of an isosceles triangle are congruent.

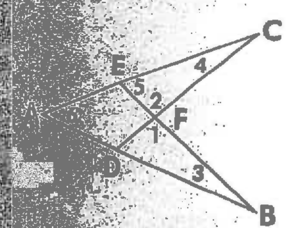
Miscellaneous Exercises

In 20–23, select the numeral preceding the expression that best completes the statement. Refer to the given figures.



Ex. 20–21

20. It can be proved that $\angle YWX \cong \angle ZXW$ if it is known that
    (1) $\overline{ZW} \cong \overline{YX}$ (2) $\overline{YW} \cong \overline{YX}$
    (3) $\overline{PW} \cong \overline{PX}$ (4) $\overline{PW} \cong \overline{YX}$
21. If $\triangle ZPW \cong \triangle YPX$, it can be proved that
    (1) $\triangle ZPW$ is isosceles (2) $\triangle YPX$ is isosceles
    (3) $\triangle PWX$ is isosceles (4) $\triangle YWX$ is isosceles



Ex. 22–23

22. If $\overline{AB} \cong \overline{AC}$, it can be proved that $\overline{CD} \cong \overline{BE}$ if it is also known that
    (1) $\angle 1 \cong \angle 2$ (2) $\angle 3 \cong \angle 4$
    (3) $\angle 3 \cong \angle 5$ (4) $\angle 4 \cong \angle 6$
23. If $\angle 3 \cong \angle 4$, it can be proved that $\overline{EC} \cong \overline{DB}$ if it is also known that
    (1) $\overline{CF} \cong \overline{BF}$ (2) $\overline{CD} \cong \overline{BE}$
    (3) $\overline{CA} \cong \overline{BA}$ (4) $\overline{EA} \cong \overline{DA}$

USING BASIC INEQUALITY POSTULATES

In an ***equality***, the left-hand member of an equation is *equal* to the right-hand member of the equation.

In an ***inequality***, the left-hand and right-hand members are not equal, usually indicated by one of two phrases: *is greater than* (>), or *is less than* (<). Examples of inequalities exist in geometry just as they exist in arithmetic and in algebra.